Formal Verification of Curved Flight Collision Avoidance Maneuvers

A Case Study

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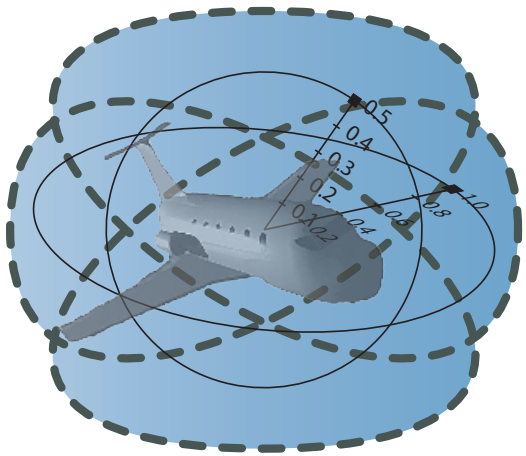
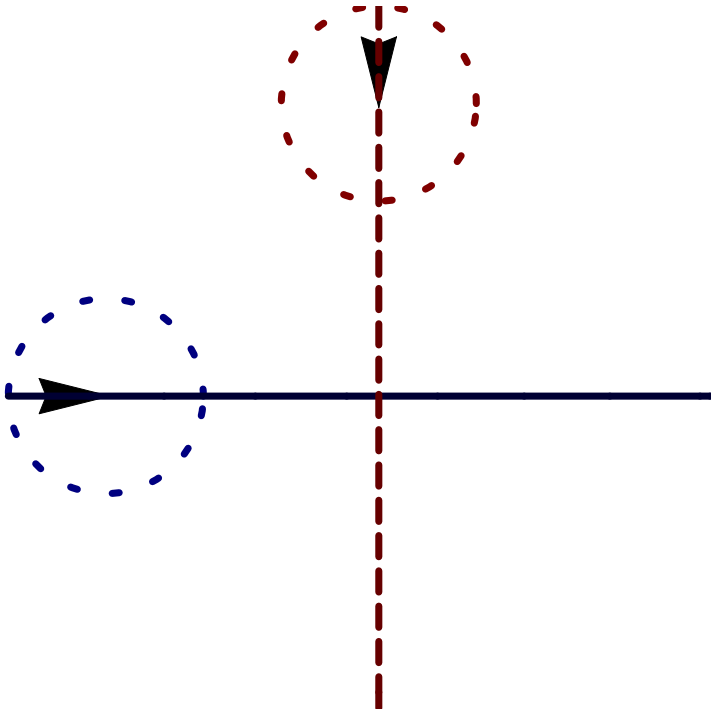
Formal Methods, FM, Eindhoven, November 2009



Outline

Air Traffic Control: Straight Lines & Instant Turns



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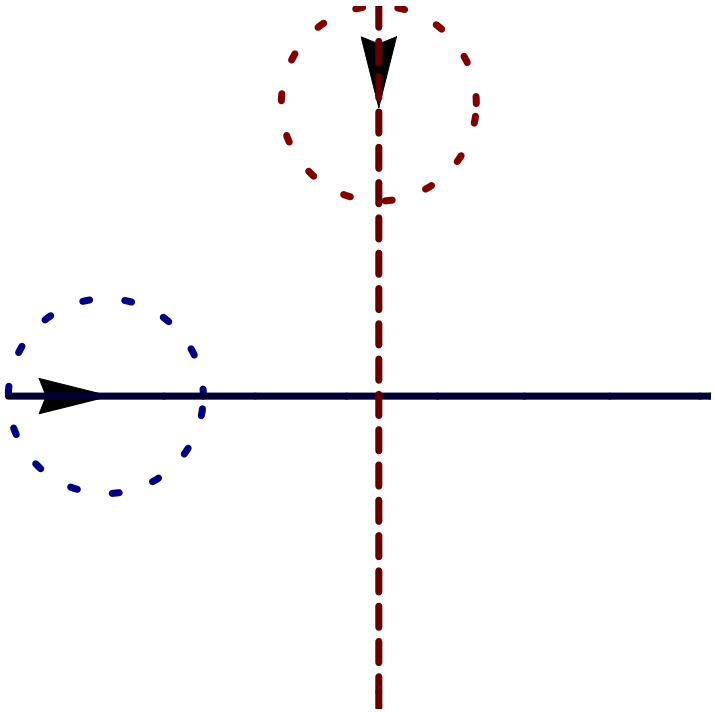

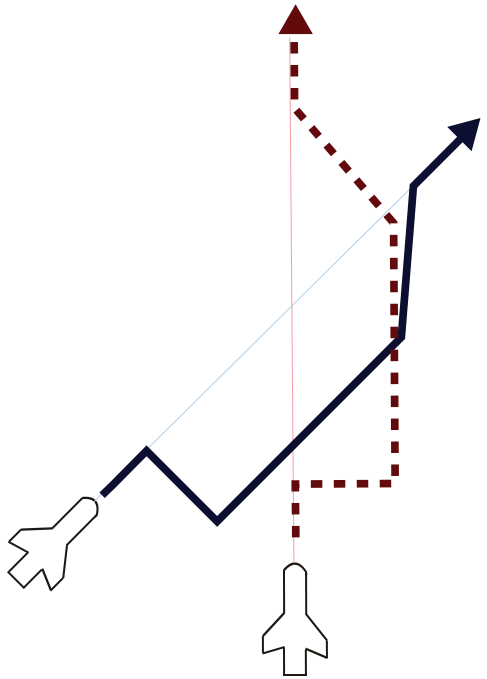

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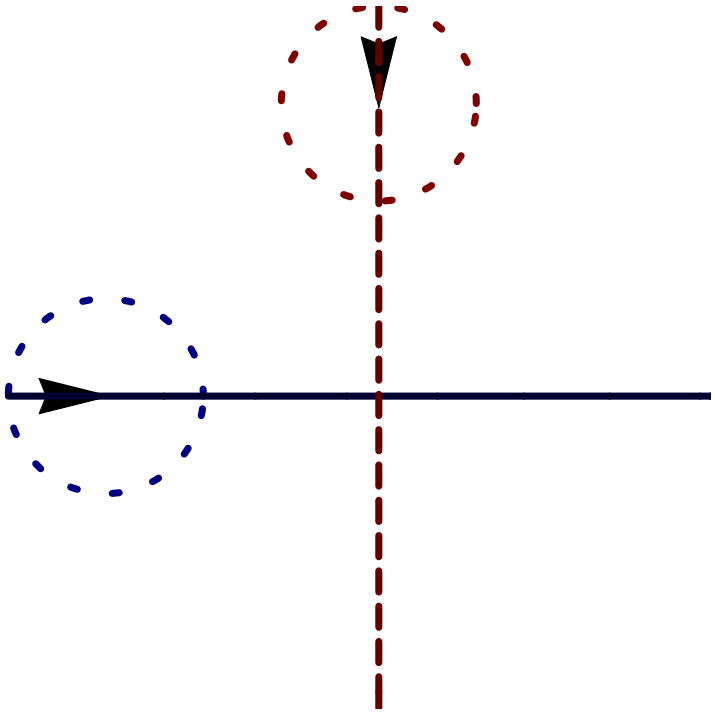
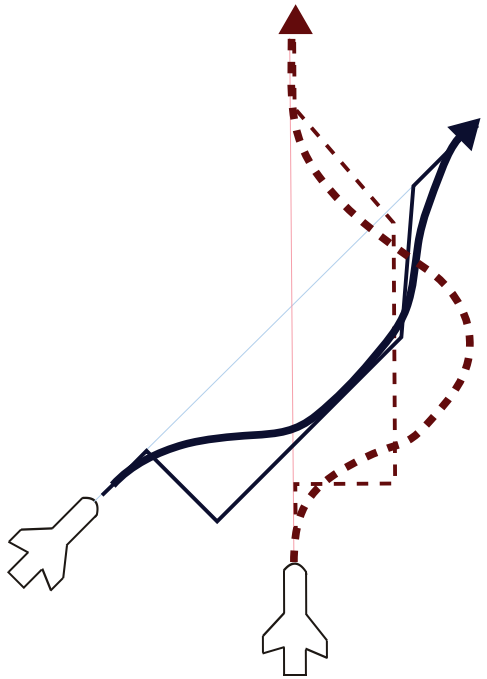
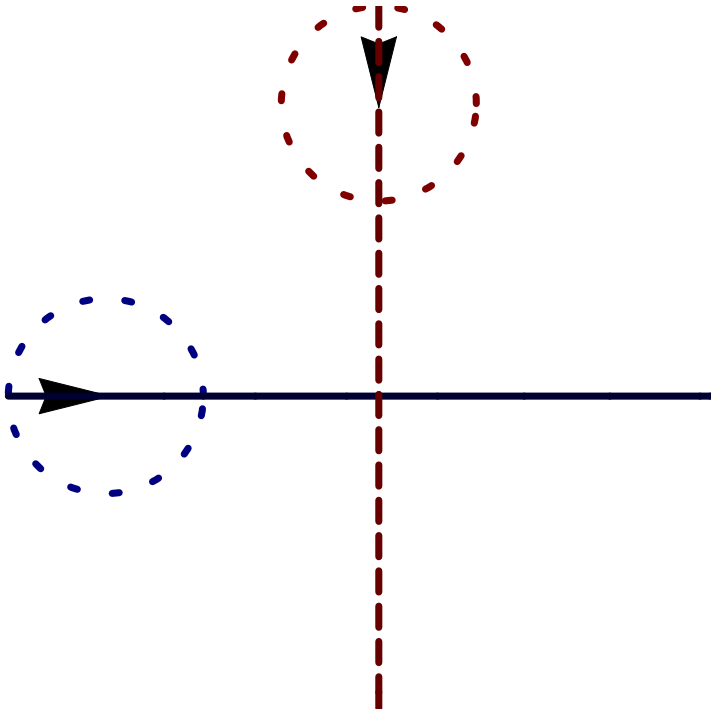




Air Traffic Control: Hybrid Systems & Curves



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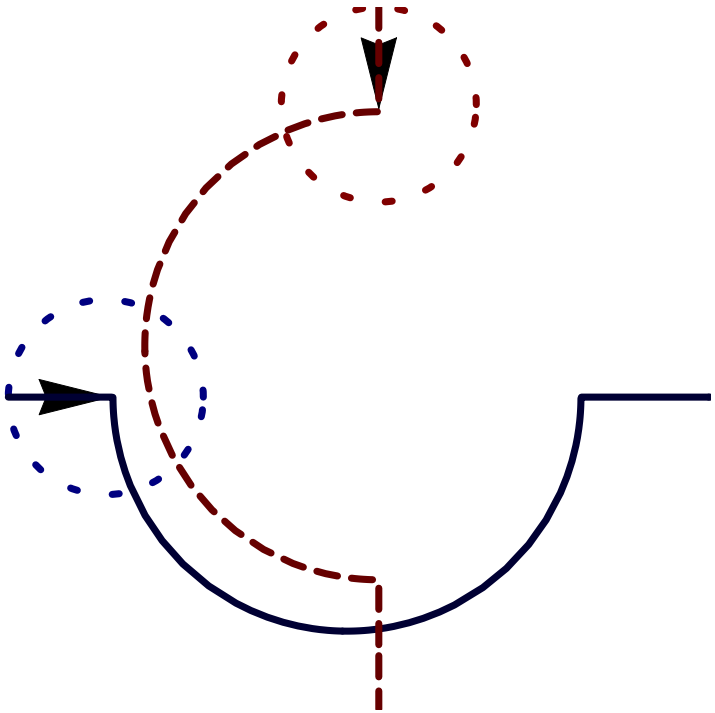

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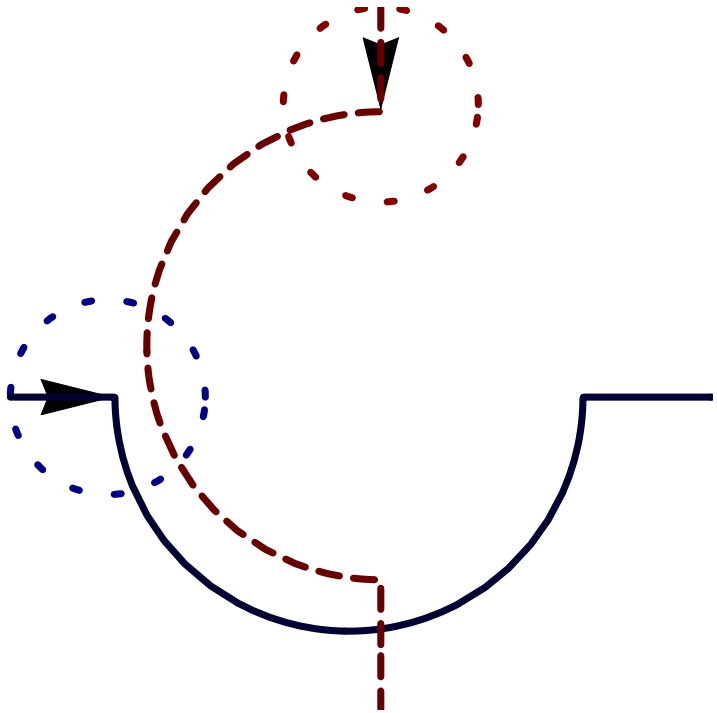

Hybrid Systems

continuous evolution along differential equations + discrete change

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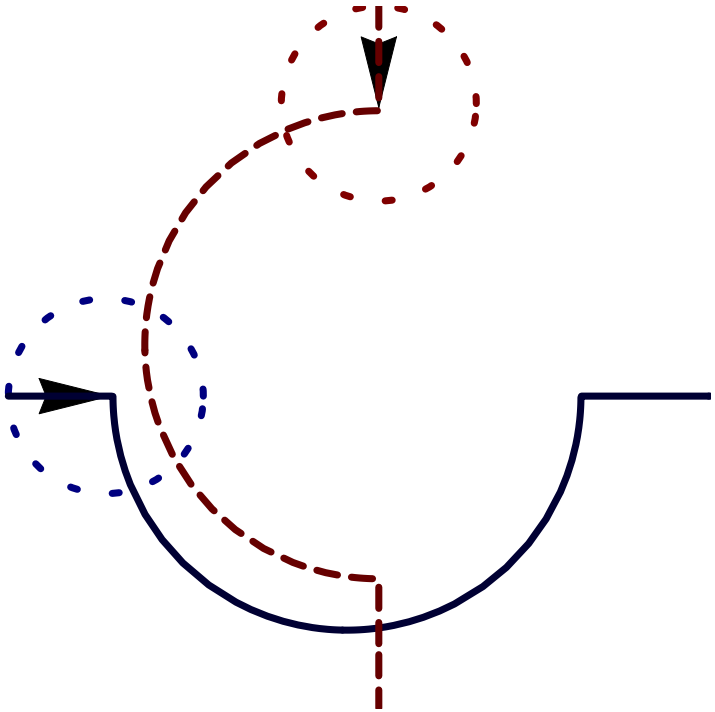

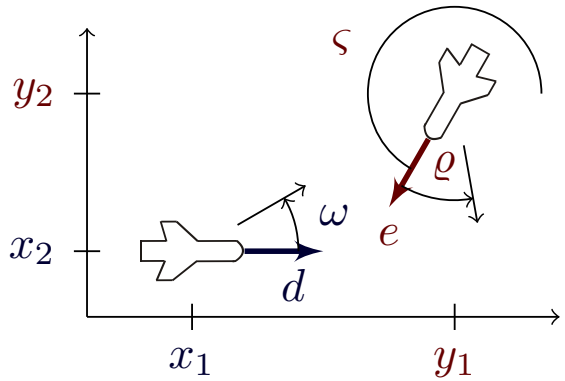

$$\begin{bmatrix} x_1' = -v_1 + v_2 \cos\vartheta + \omega x_2 \\ x_2' = v_2 \sin\vartheta - \omega x_1 \\ \vartheta' = \varrho - \omega \end{bmatrix}$$

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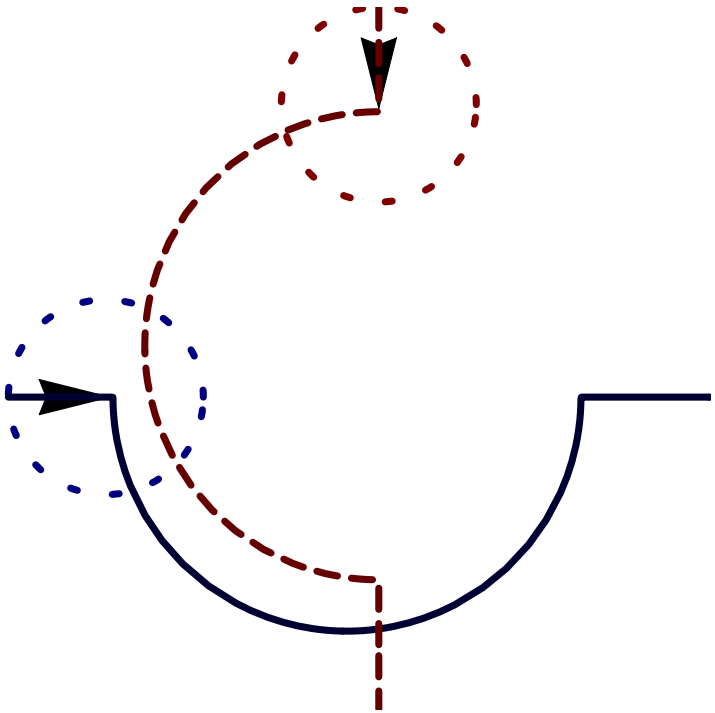

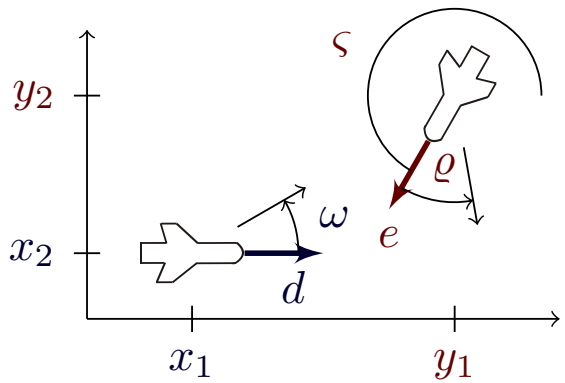

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Example ("Solving" differential equations)

$$x_1(t) = \frac{1}{\omega\varrho}\big(x_1\omega\varrho\cos t\omega - v_2\omega\cos t\omega\sin\vartheta + v_2\omega\cos t\omega\cos t\varrho\sin\vartheta - v_1\varrho\sin t\omega$$
$$+ x_2\omega\varrho\sin t\omega - v_2\omega\cos\vartheta\cos t\varrho\sin t\omega - v_2\omega\sqrt{1-\sin\vartheta^2}\sin t\omega$$
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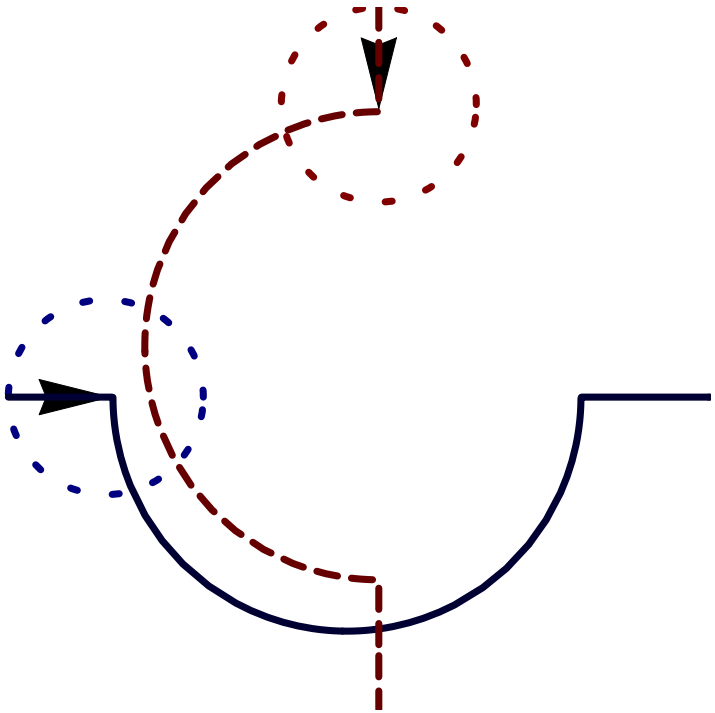

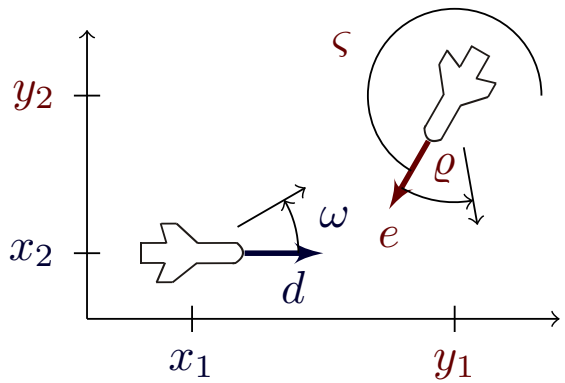

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$$\forall t \geq 0 \quad \frac{1}{\omega\varrho}\big(x_1\omega\varrho\cos t\omega - v_2\omega\cos t\omega\sin\vartheta + v_2\omega\cos t\omega\cos t\varrho\sin\vartheta - v_1\varrho\sin t\omega$$
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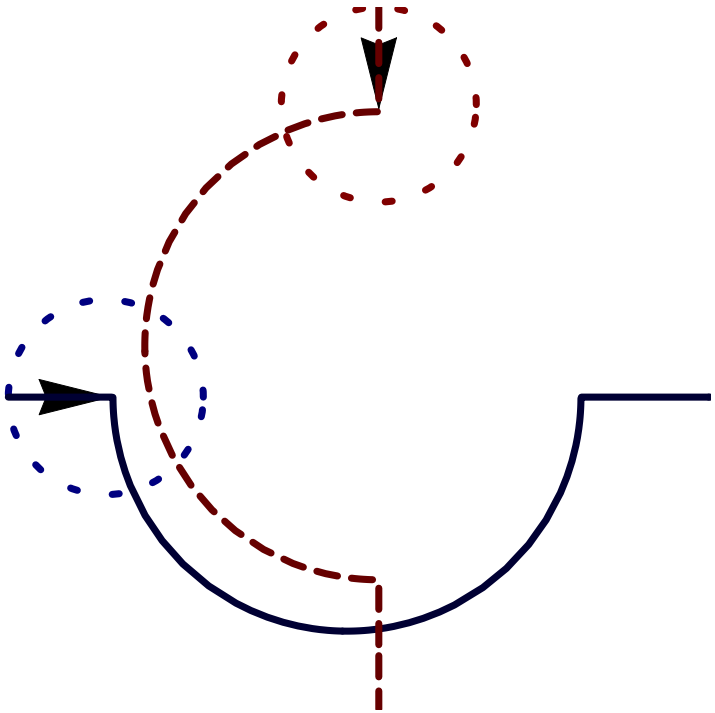

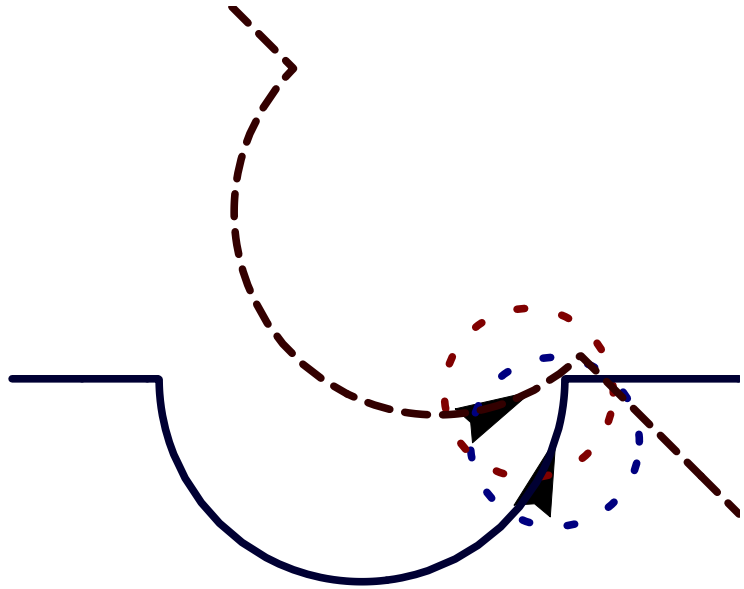

Hybrid Systems

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Introduce: Flyable Roundabout Maneuver

Problem $\Rightarrow$ Solution

- Unrealistic instant turns can cause problems

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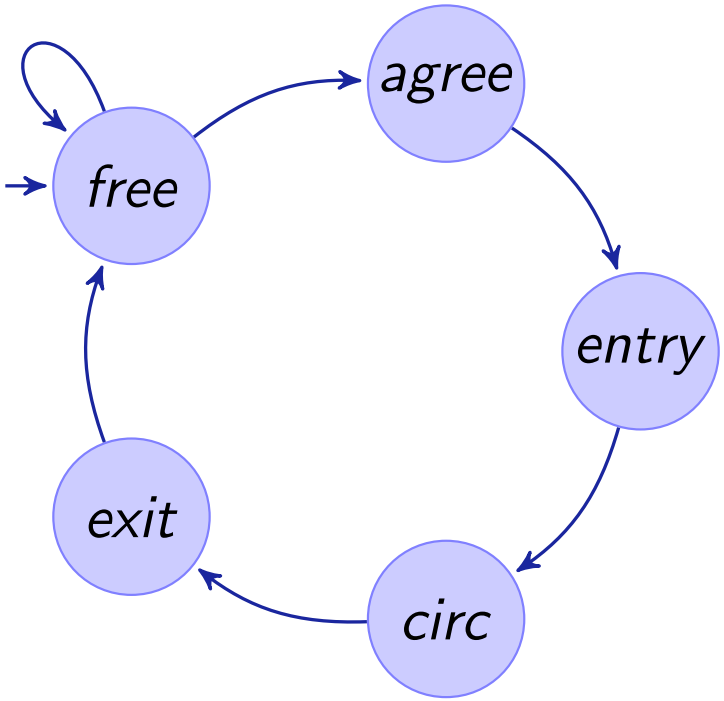

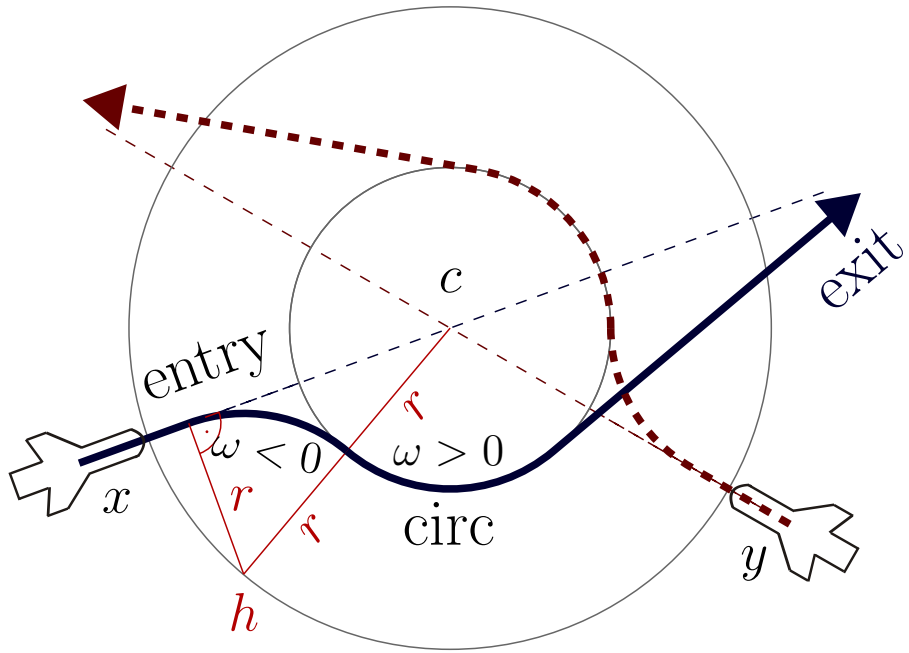

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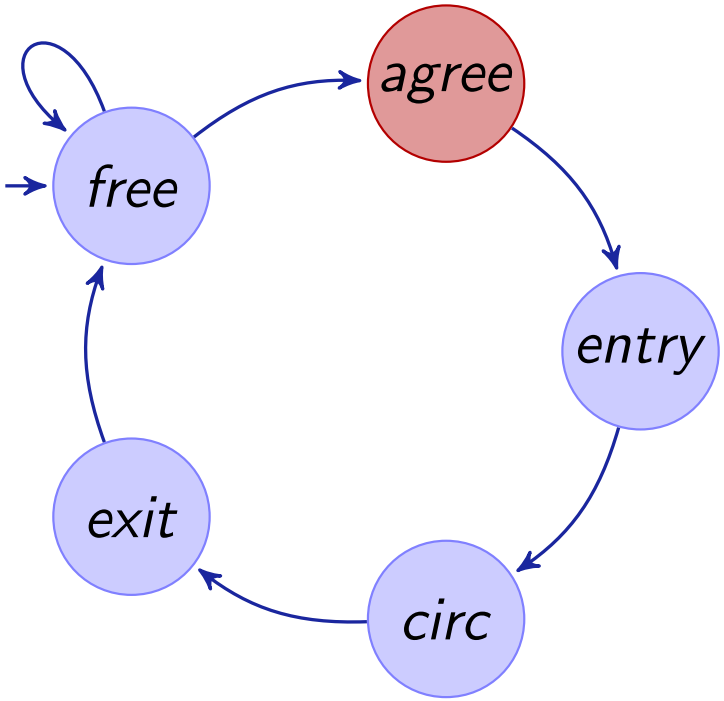

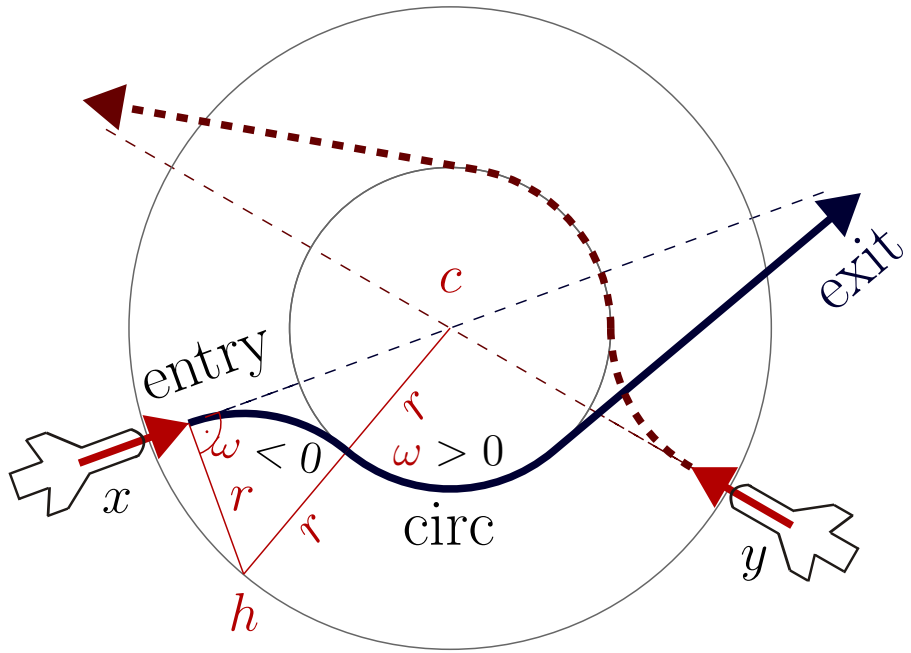

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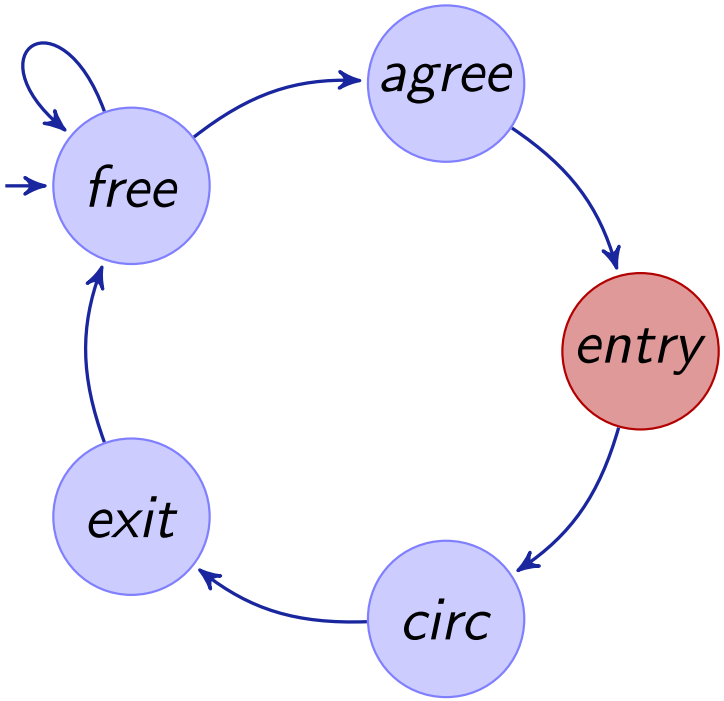

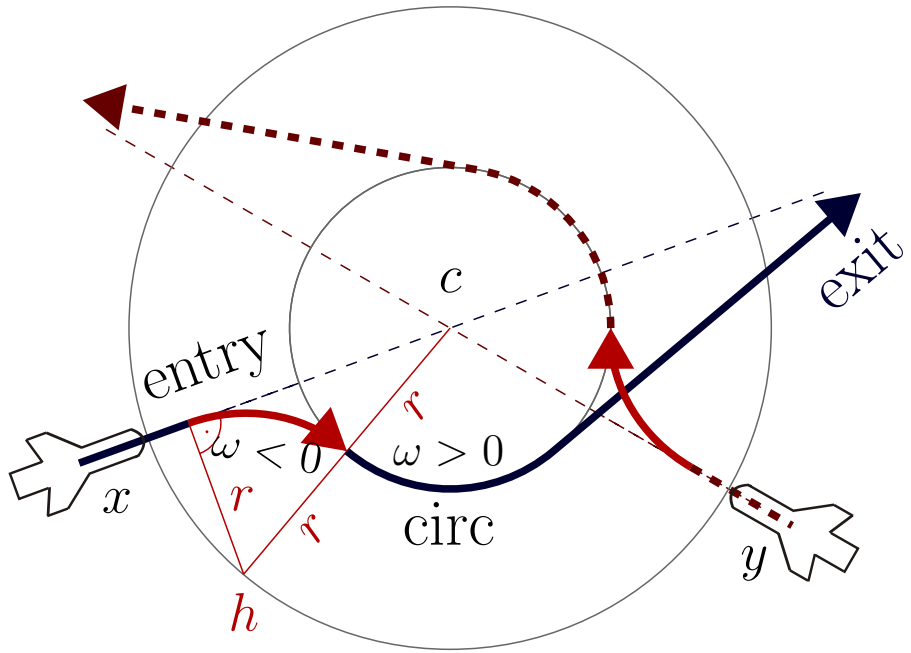

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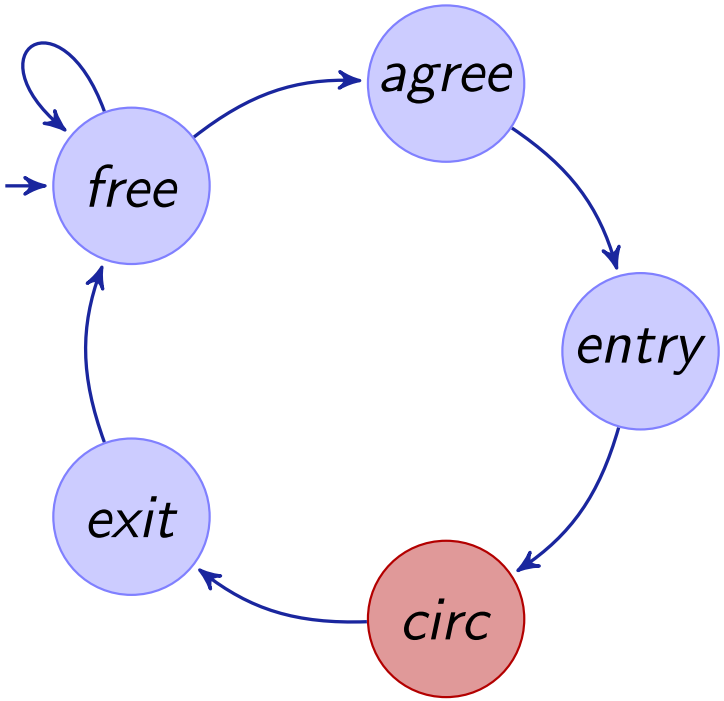

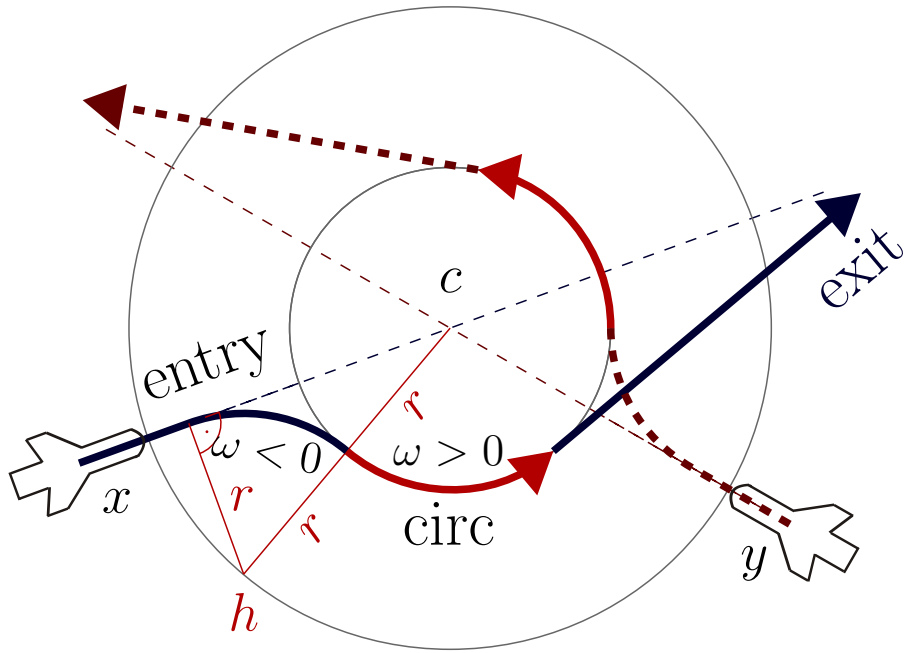

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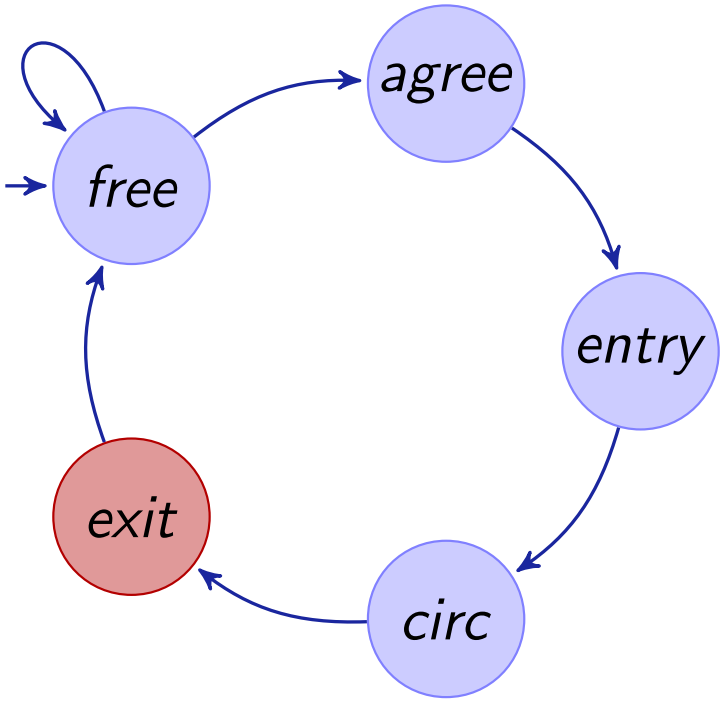

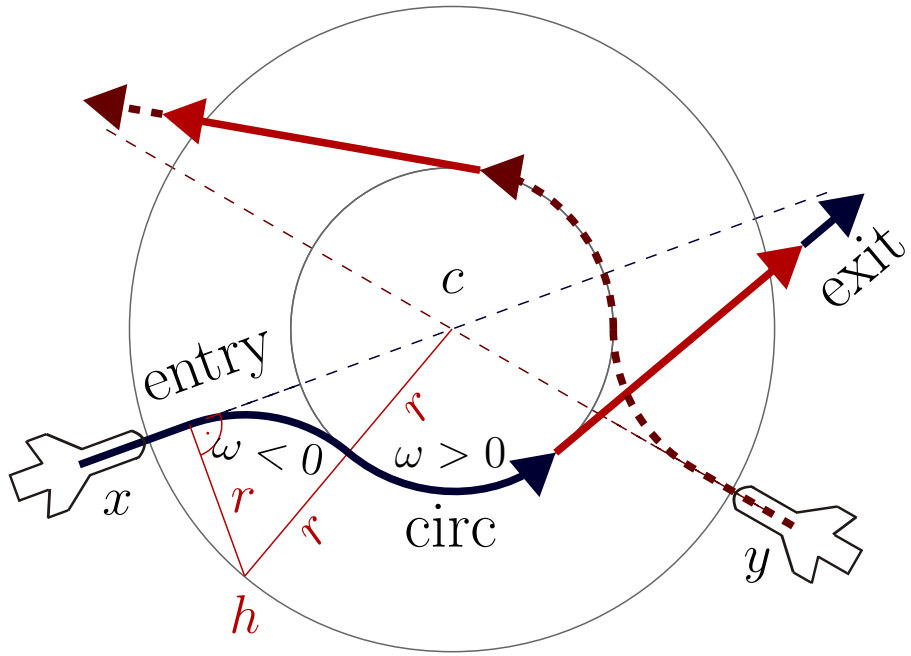

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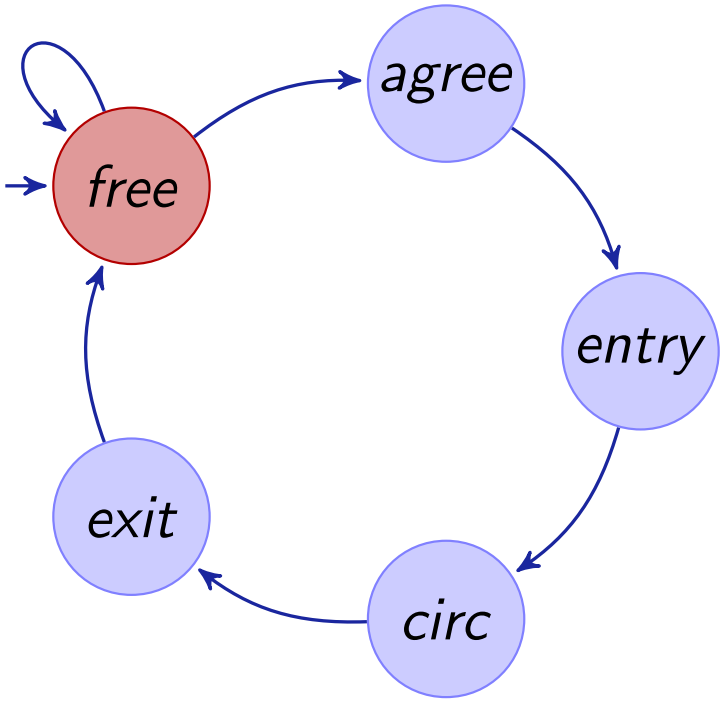

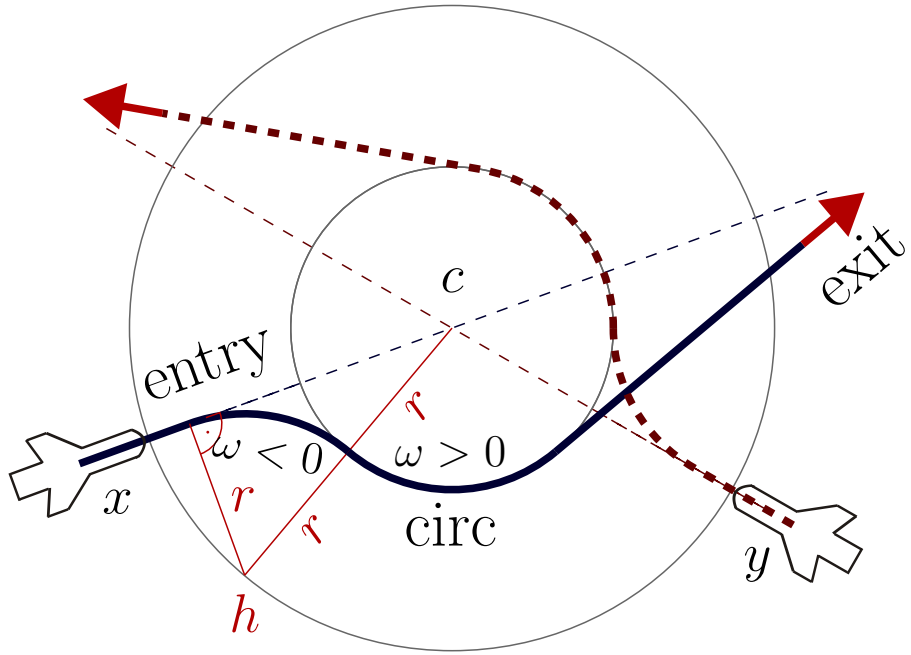

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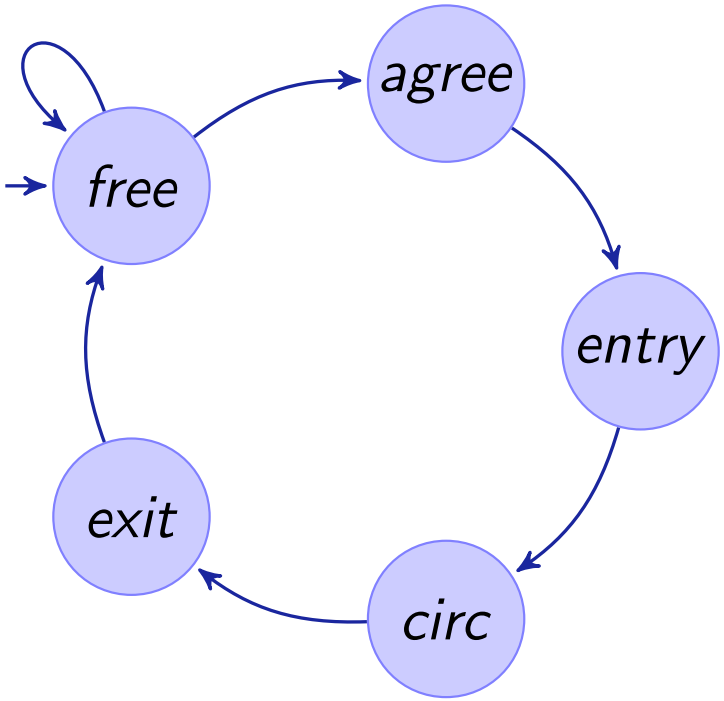

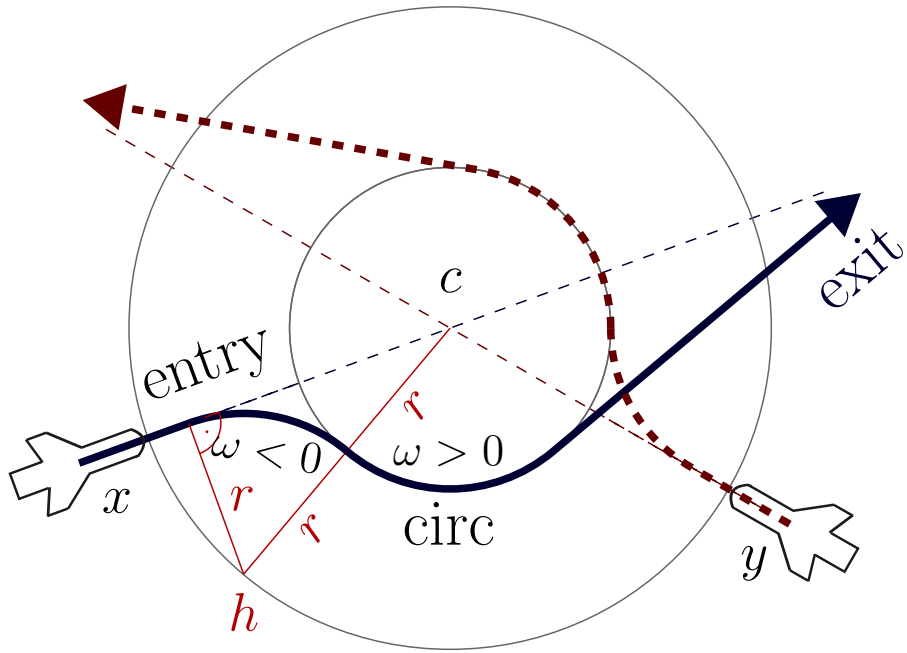

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Verification for: nonlinear curve dynamics + mode switching?

Outline

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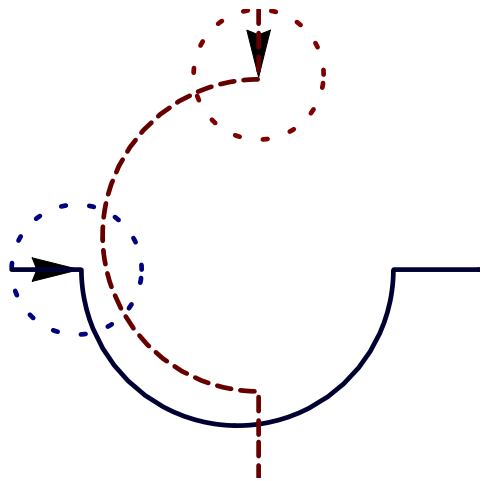
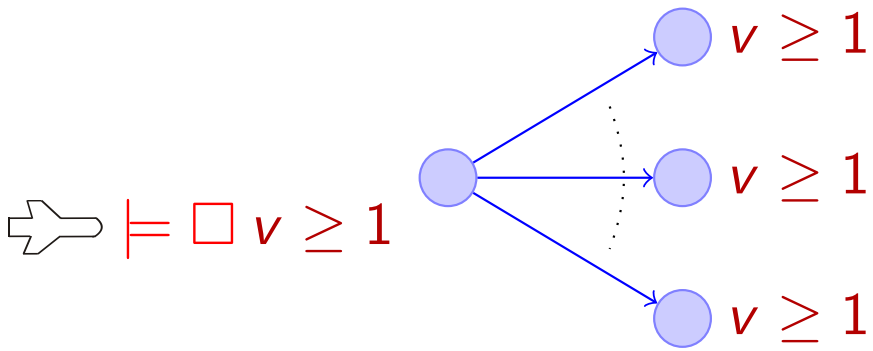
Differential Dynamic Logic for Hybrid Programs

differential dynamic logic

$d\mathcal{L} = FOL_{\mathbb{R}} + ML$





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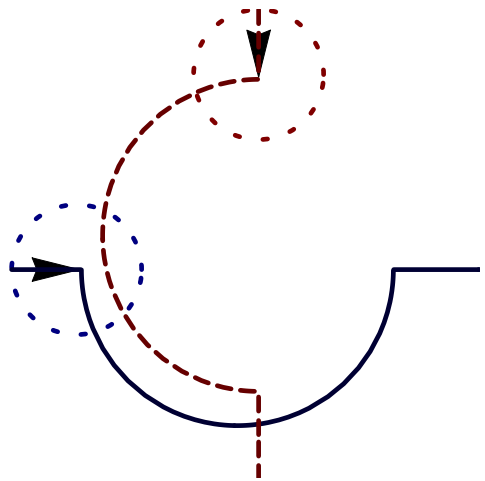
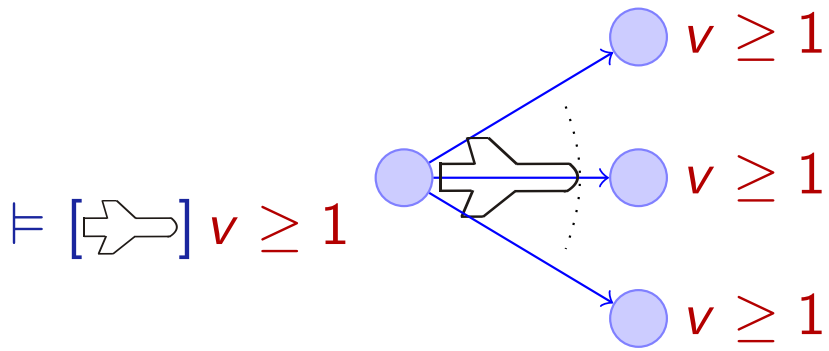
differential dynamic logic

$\mathrm{d}\mathcal{L} = \mathrm{FOL}_{\mathbb{R}} + \mathrm{DL}$





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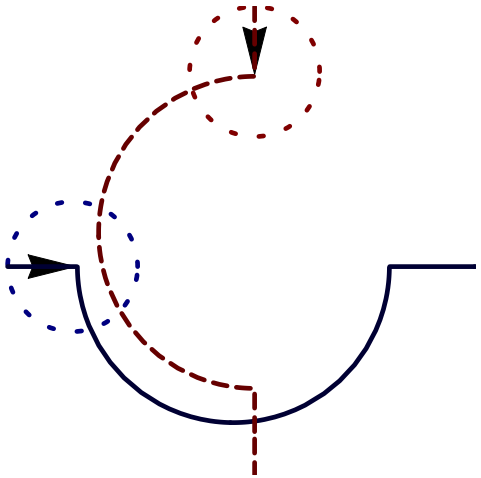
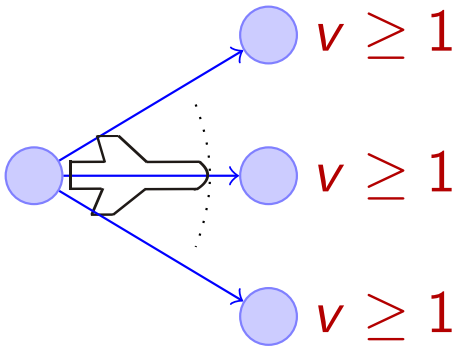


$$\vDash [d_1' = -\omega d_2, d_2' = \omega d_1]\, v \geq 1$$



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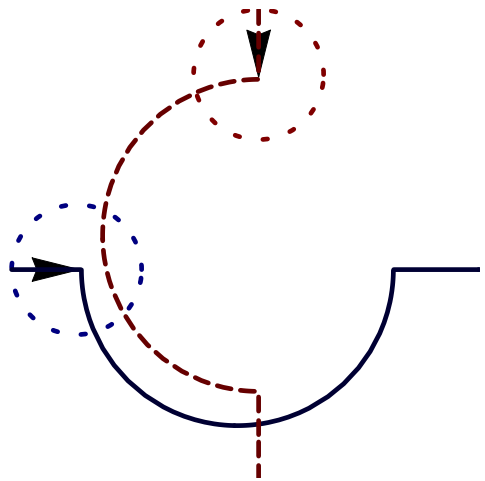
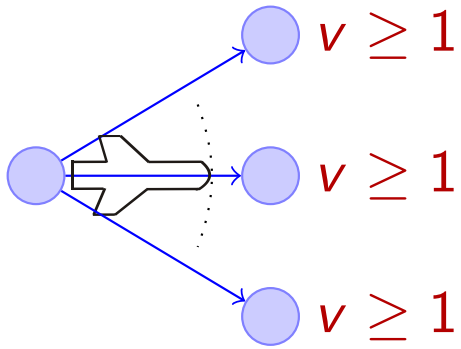


$$\vDash [\texttt{if}(x_1 > 0)\, \omega := 1;\ d_1' = -\omega d_2, d_2' = \omega d_1]\, v \geq 1$$



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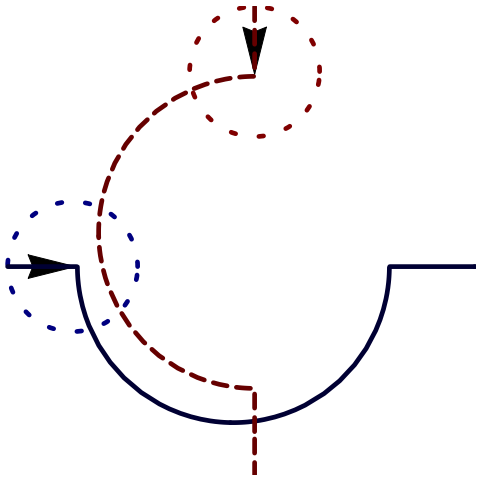
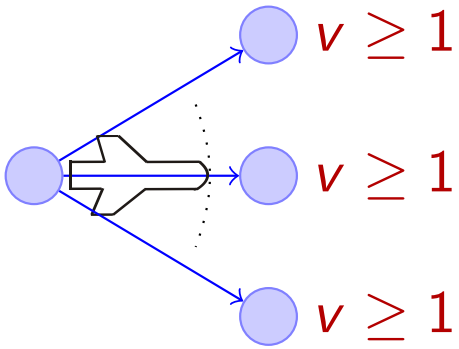


$$\vDash [\underbrace{\texttt{if}(x_1 > 0)\, \omega := 1;\ d_1' = -\omega d_2, d_2' = \omega d_1}_{\text{hybrid program}}]\, v \geq 1$$



Differential Dynamic Logic for Hybrid Programs

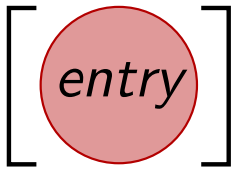
Definition (d$\mathcal{L}$ Formula ϕ)

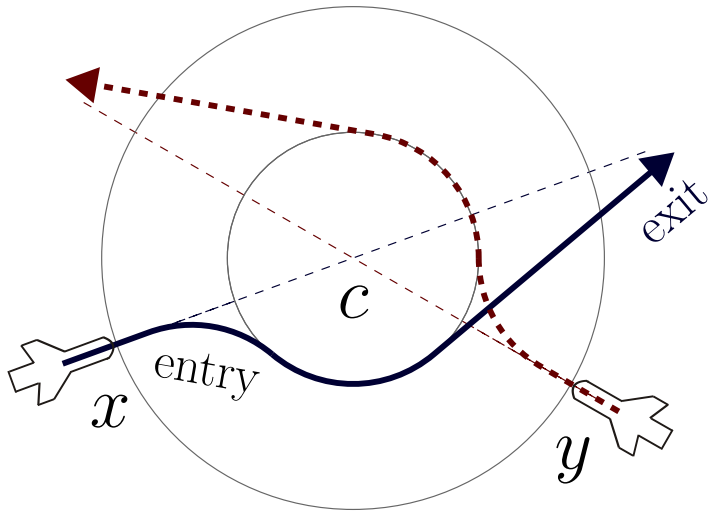
$$\theta_1 \geq \theta_2 \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid \phi \rightarrow \psi \mid \forall x\, \phi \mid \exists x\, \phi \mid [\alpha]\phi \mid \langle\alpha\rangle\phi$$

with terms θ_1, θ_2 of nonlinear real arithmetic $(+, \cdot)$

Definition (Hybrid program α)

$x' = f(x) \wedge H$	(continuous evolution)	
$x := f(x)$	(discrete jump)	jump & test
$?H$	(conditional execution)	jump & test
$\alpha; \beta$	(seq. composition)	Kleene algebra
$\alpha \cup \beta$	(nondet. choice)	Kleene algebra
α^*	(nondet. repetition)	Kleene algebra



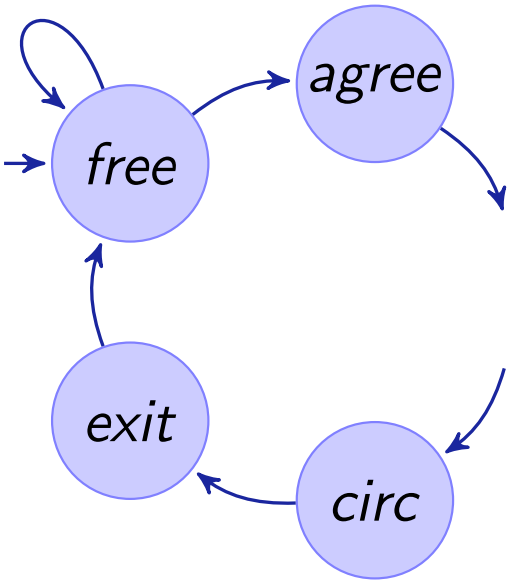


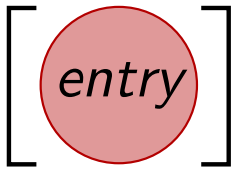
Example

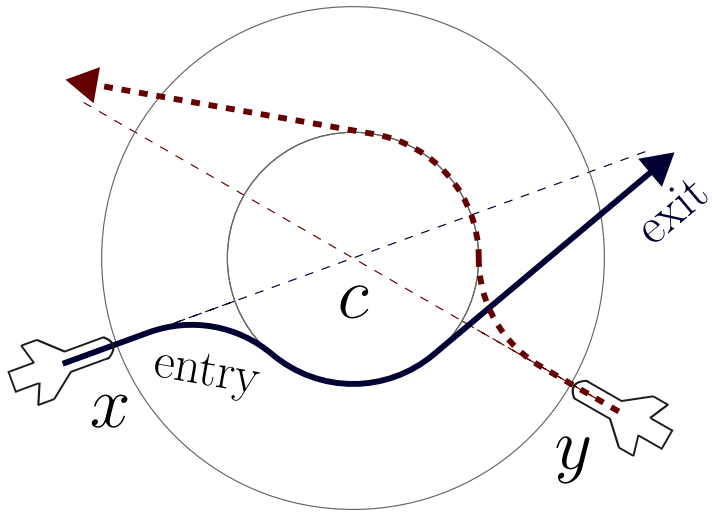
$$safe \wedge far \rightarrow [entry](safe \wedge tangential)$$

where $safe \equiv (x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2$

Differential Dynamic Logic for Compositional Verification



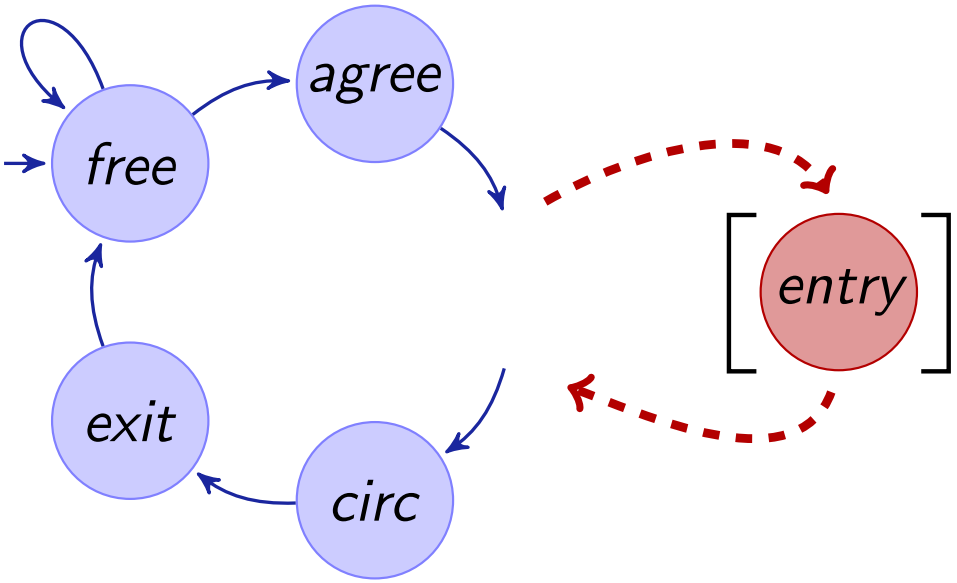


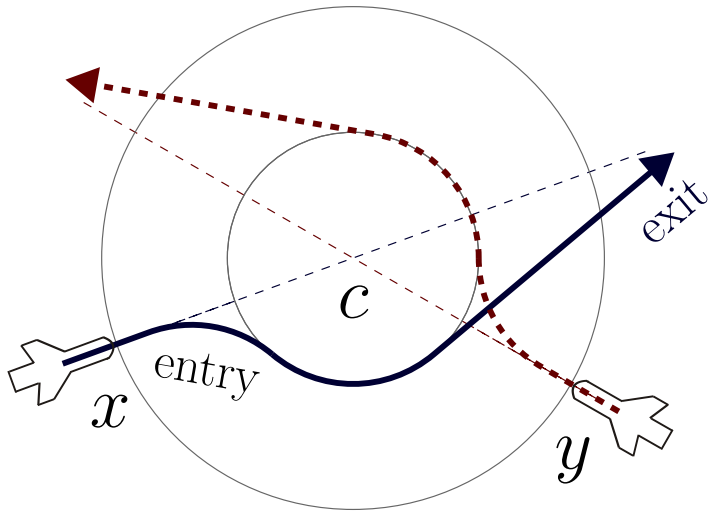


Example

$$
\begin{aligned}
\mathit{safe} \wedge \mathit{far} &\rightarrow [\mathit{entry}](\mathit{safe} \wedge \mathit{tangential}) \\
\mathit{safe} \wedge \mathit{tangential} &\rightarrow [\mathit{other\ subsystem}]\mathit{safe} \\
\text{where } \mathit{safe} &\equiv (x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2
\end{aligned}
$$

Differential Dynamic Logic for Compositional Verification





Example

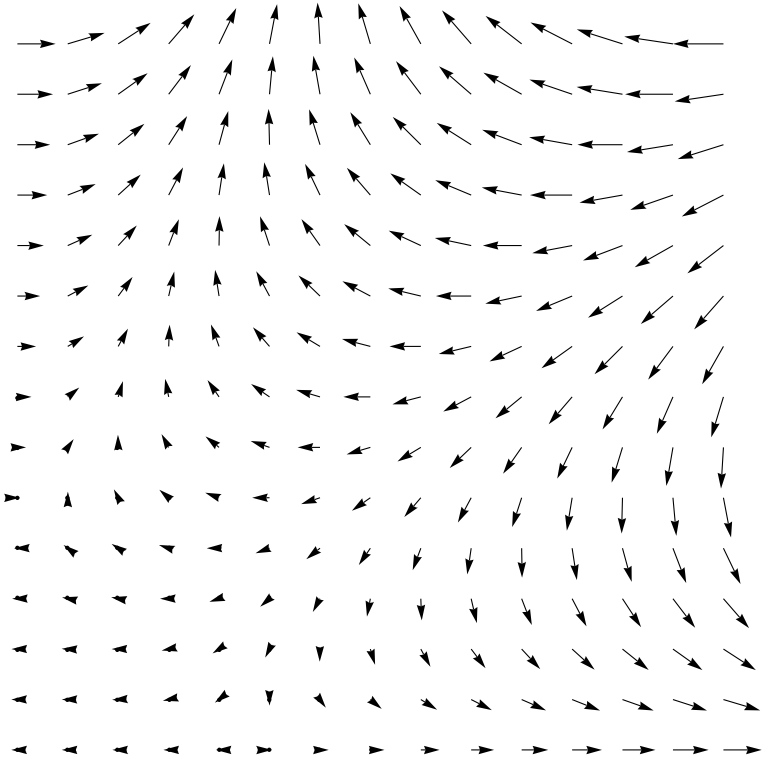
$$\left.\begin{array}{rcl} \mathit{safe} \wedge \mathit{far} & \rightarrow & [\mathit{entry}](\mathit{safe} \wedge \mathit{tangential}) \\ \mathit{safe} \wedge \mathit{tangential} & \rightarrow & [\mathit{other\ subsystem}]\mathit{safe} \end{array}\right\} \text{conjunction}$$

$$\text{where} \quad \mathit{safe} \equiv (x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2$$

Differential Invariants for Differential Equations

"Definition" (Differential Invariant)

"Formula that remains true in the direction of the dynamics"



Differential Invariants for Differential Equations

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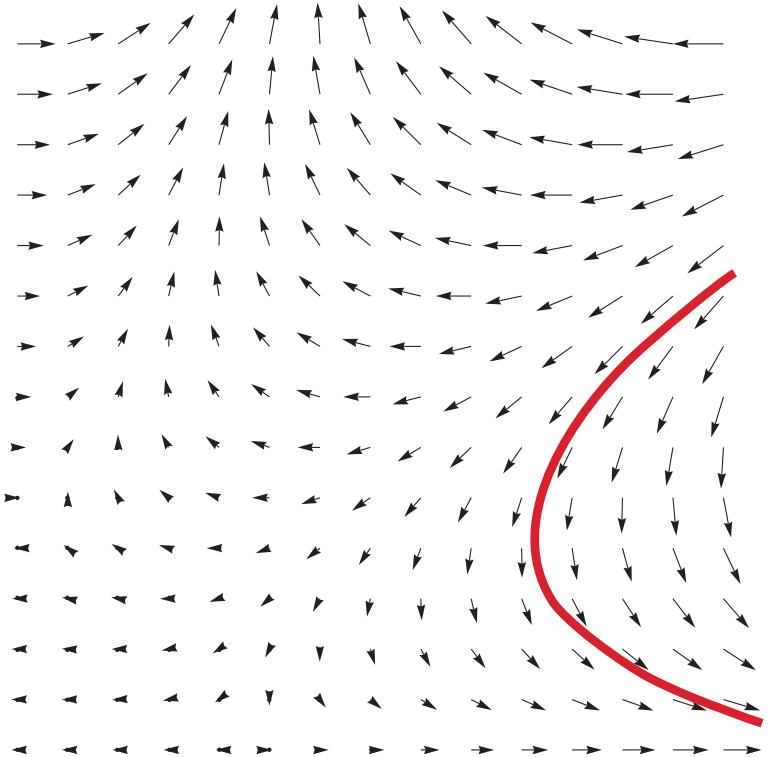

Differential Invariants for Differential Equations

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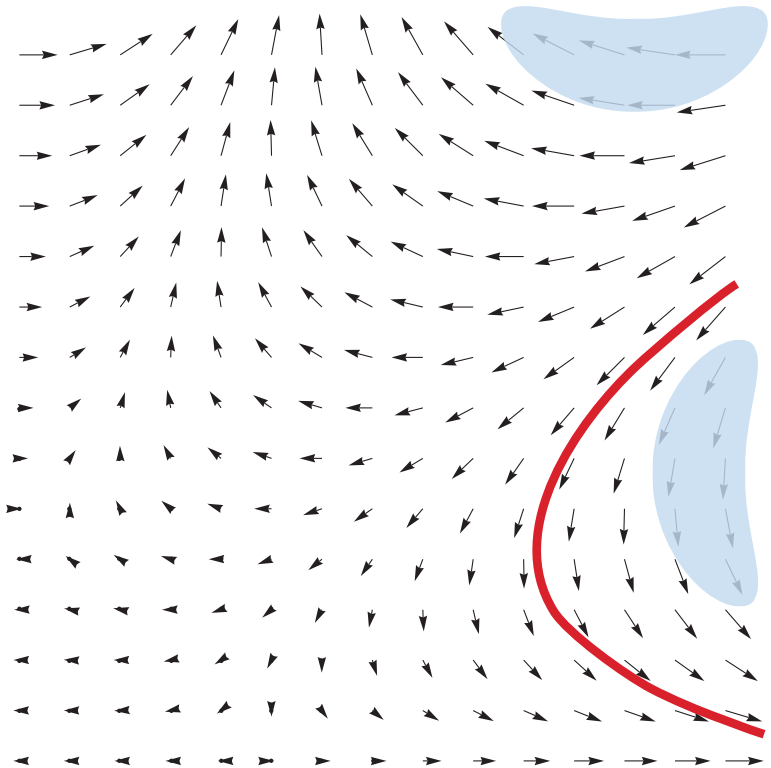
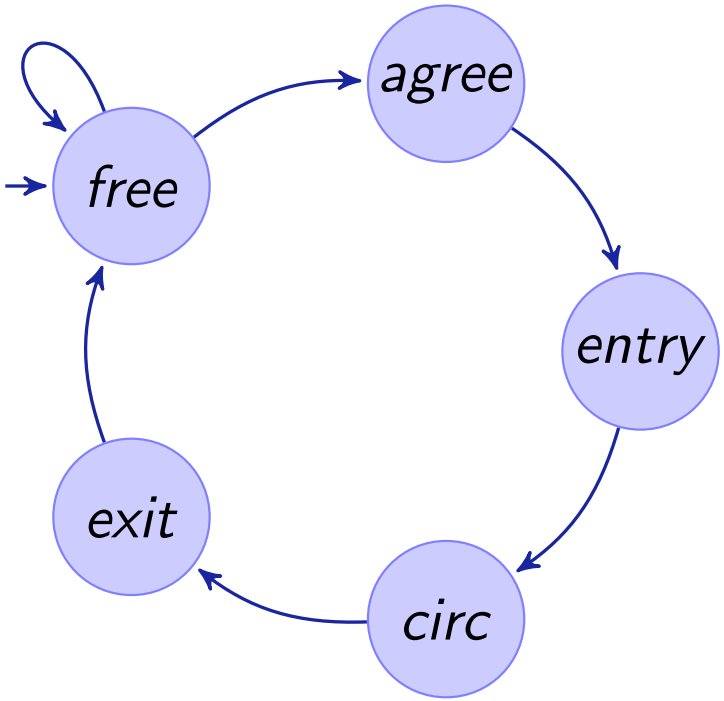
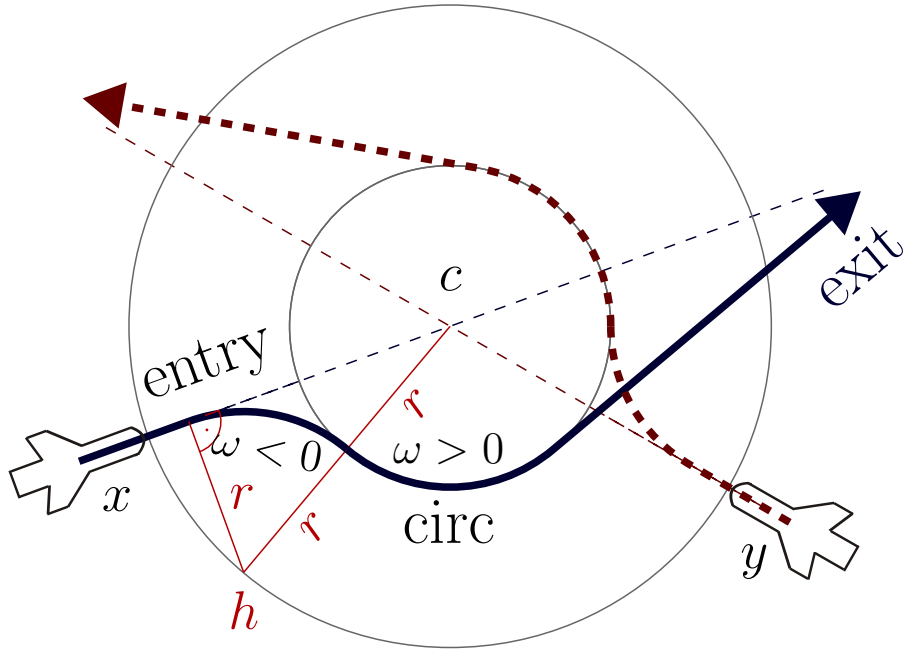


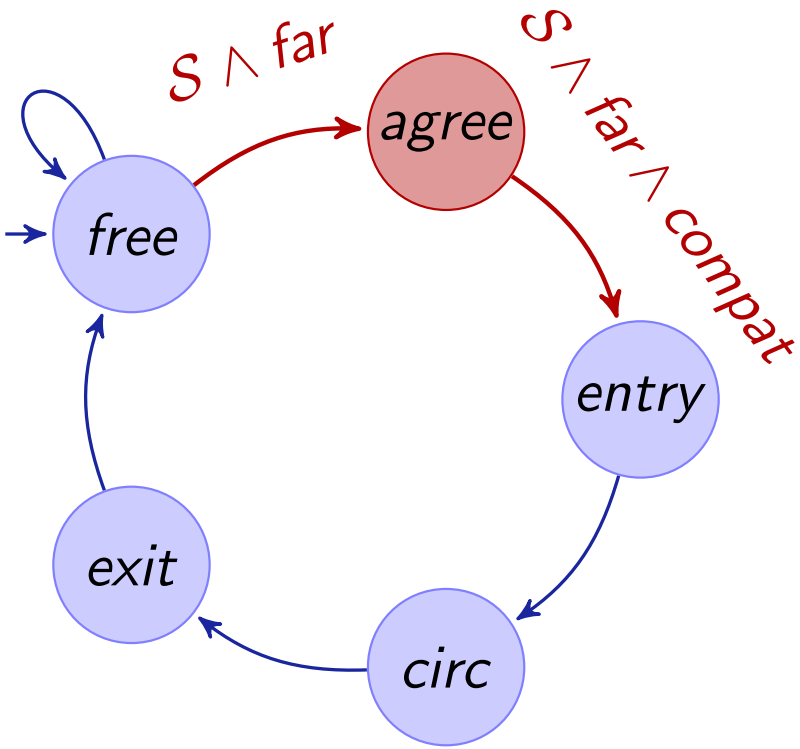
Outline

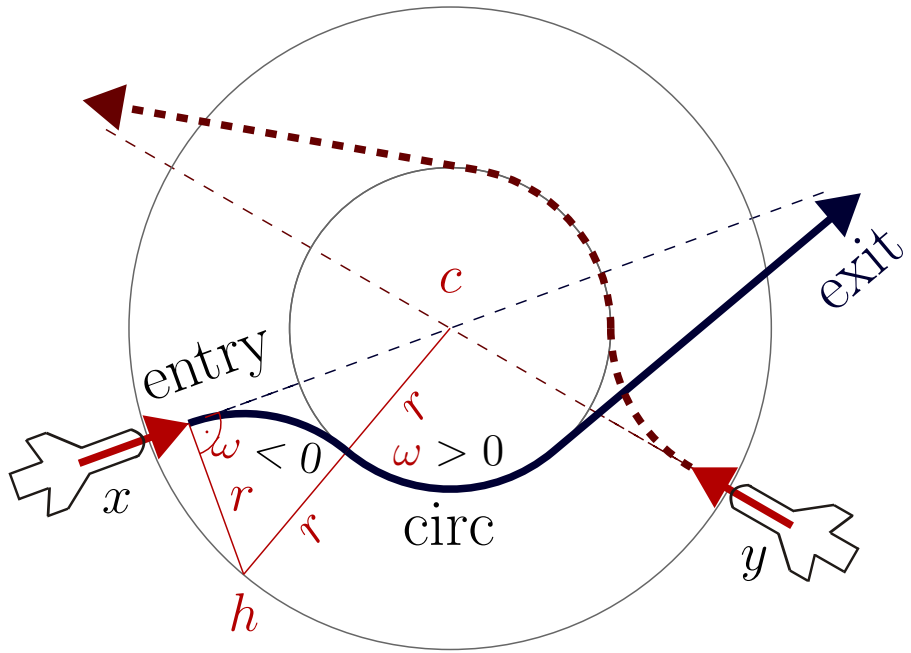
Verification Loop for Air Traffic Control





Verification Loop for Air Traffic Control

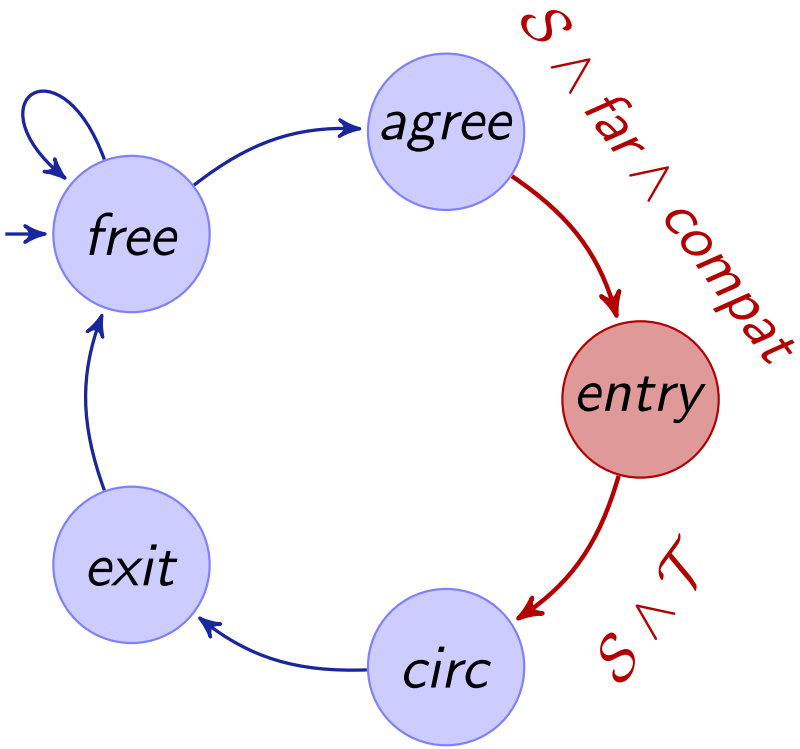


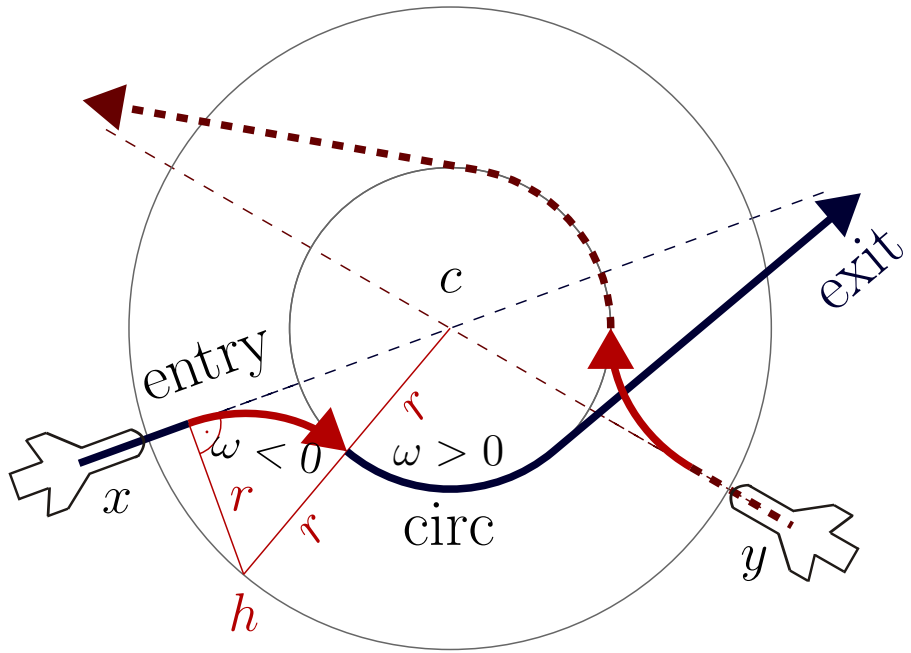


Example (d$\mathcal{L}$ formula of verification subgoal)

$$safe \wedge far \rightarrow [agree](safe \wedge far \wedge compatible)$$

Verification Loop for Air Traffic Control

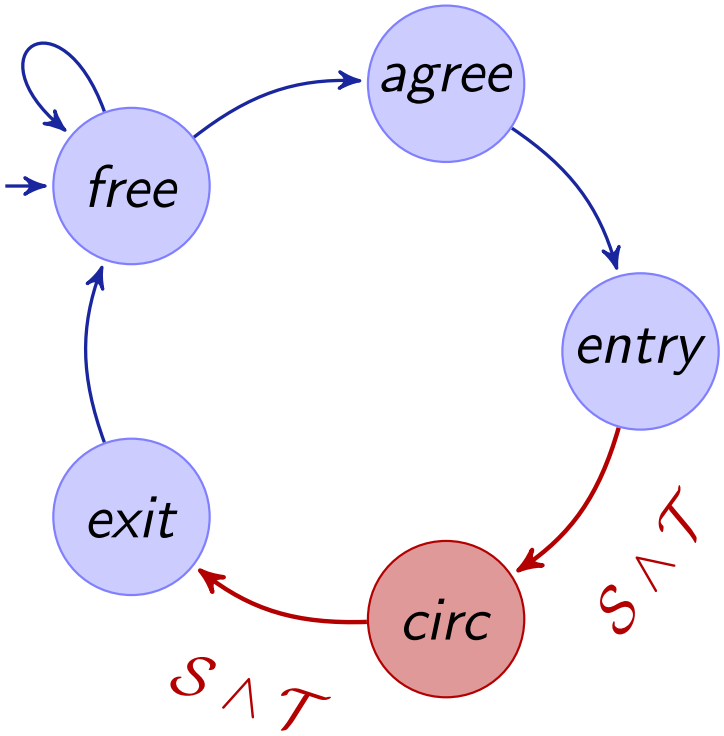


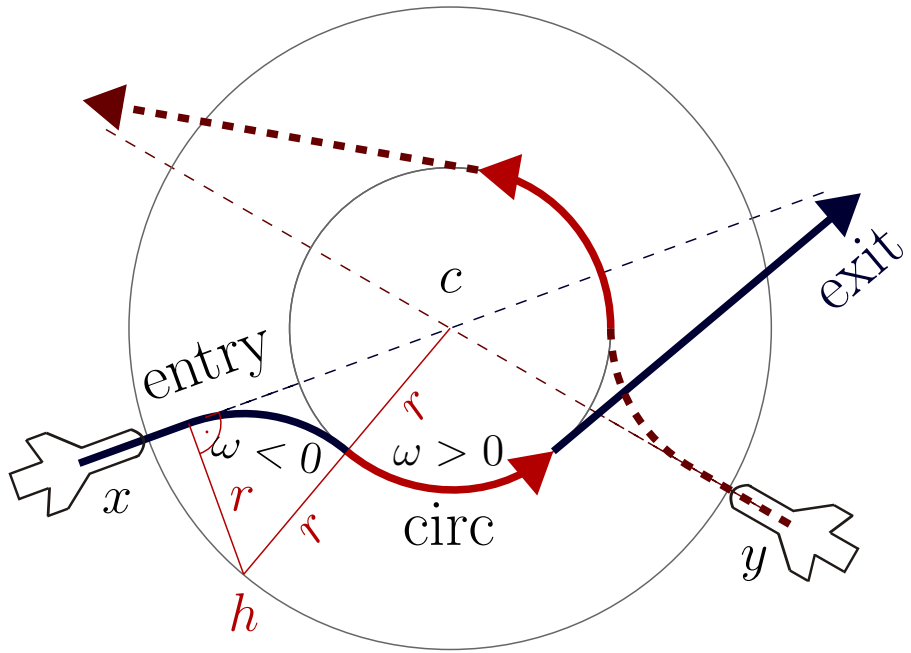


Example (d$\mathcal{L}$ formula of verification subgoal)

$$safe \wedge far \wedge compatible \rightarrow [entry](safe \wedge tangential)$$

Verification Loop for Air Traffic Control

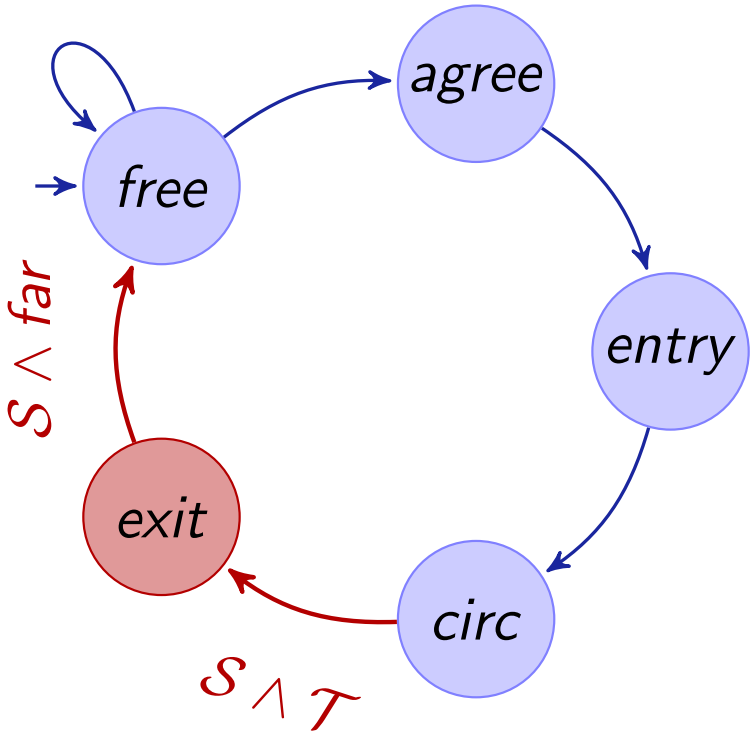


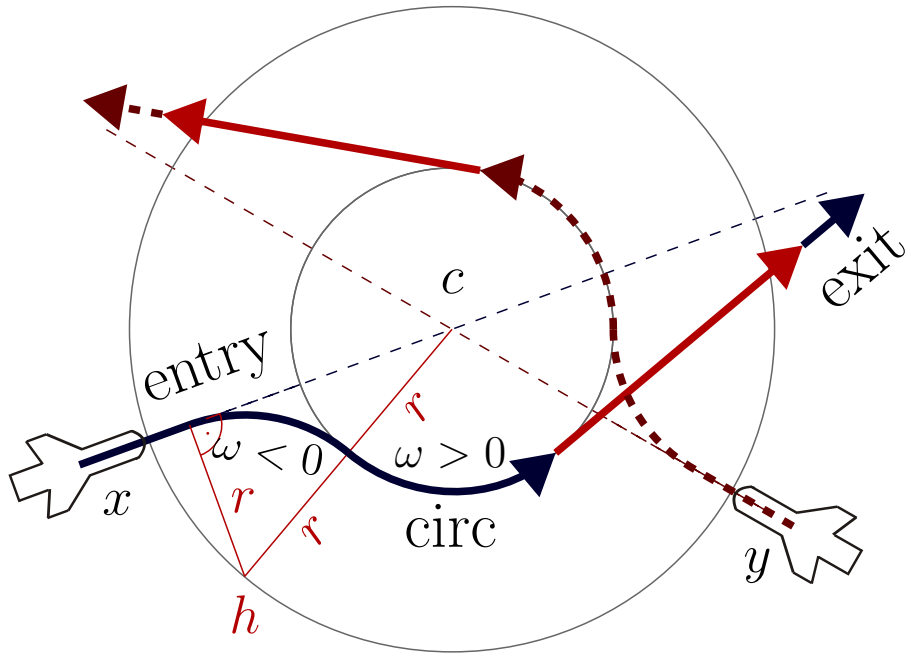


Example (d$\mathcal{L}$ formula of verification subgoal)

$$safe \wedge tangential \rightarrow [circ](safe \wedge tangential)$$

Verification Loop for Air Traffic Control

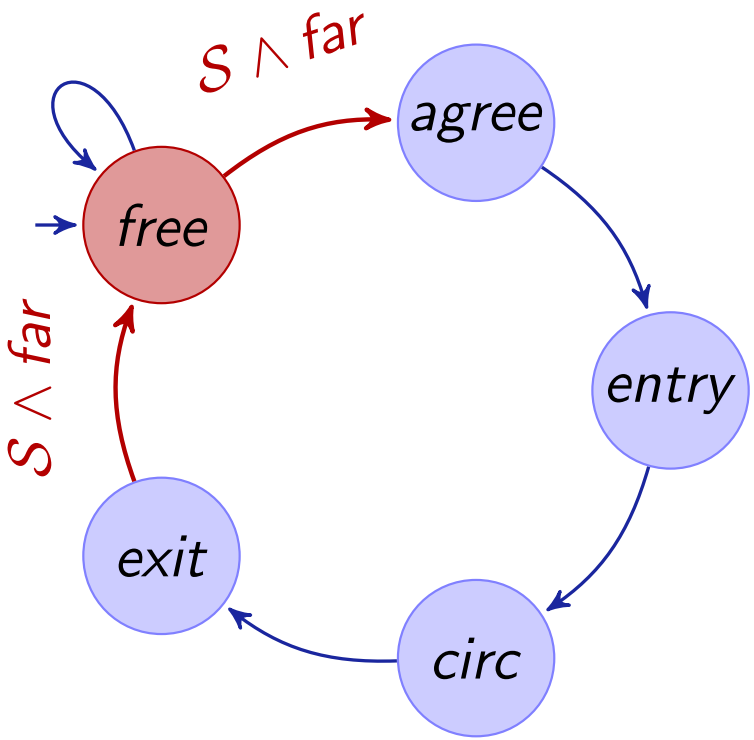


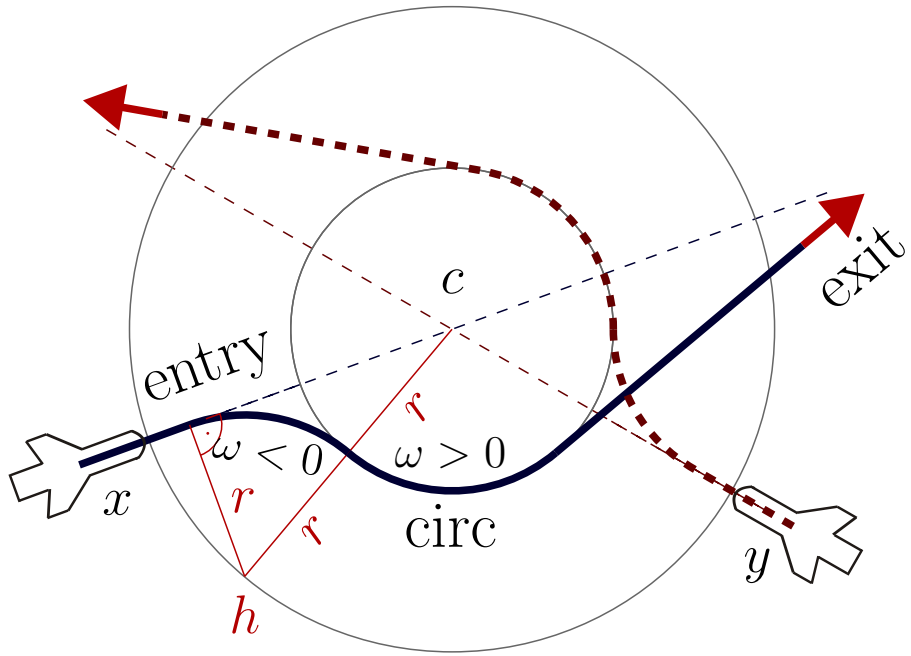


Example (d$\mathcal{L}$ formula of verification subgoal)

$$safe \wedge tangential \rightarrow [exit](safe \wedge far)$$

Verification Loop for Air Traffic Control

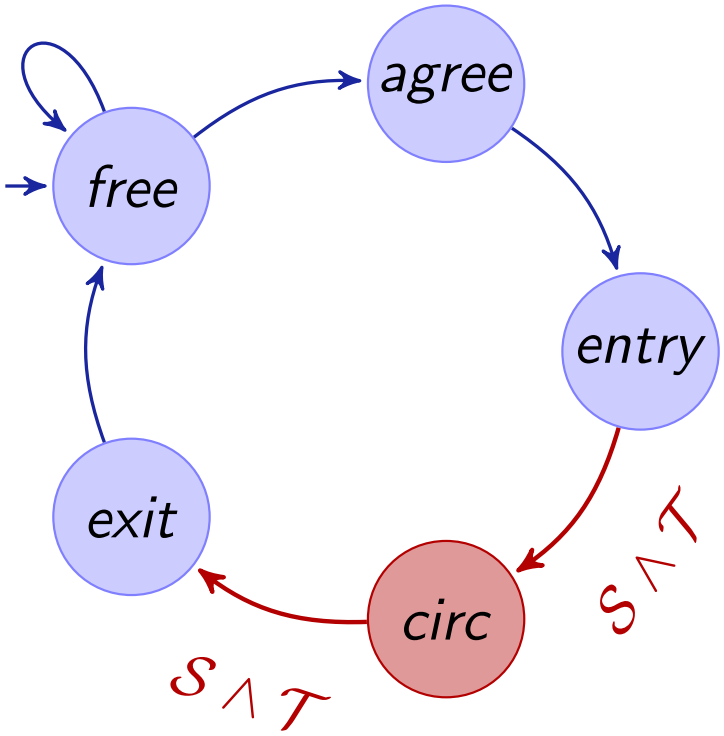


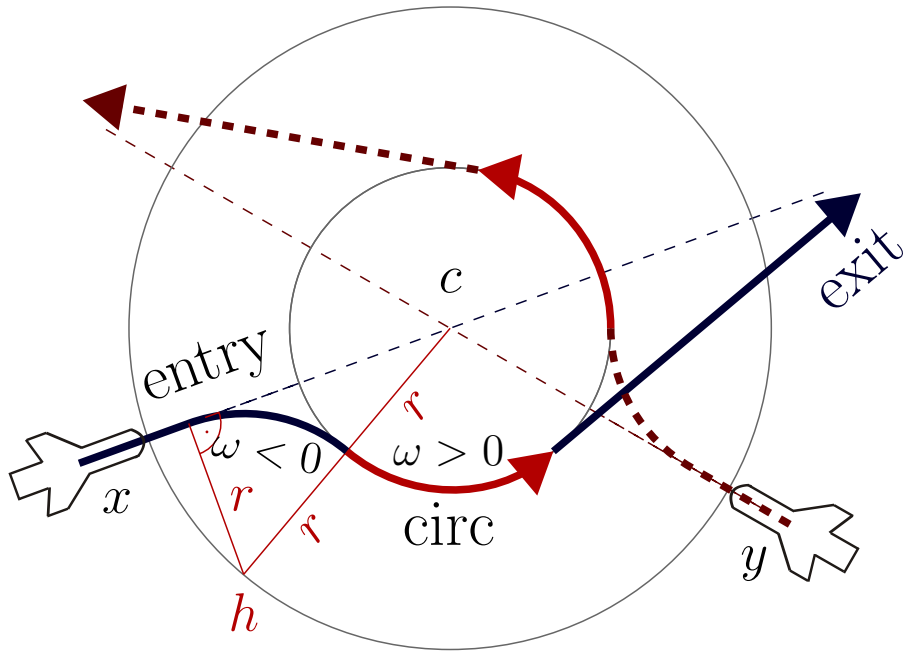


Example (d$\mathcal{L}$ formula of verification subgoal)

$$safe \wedge far \rightarrow [free](safe \wedge far)$$

Verification Loop for Air Traffic Control



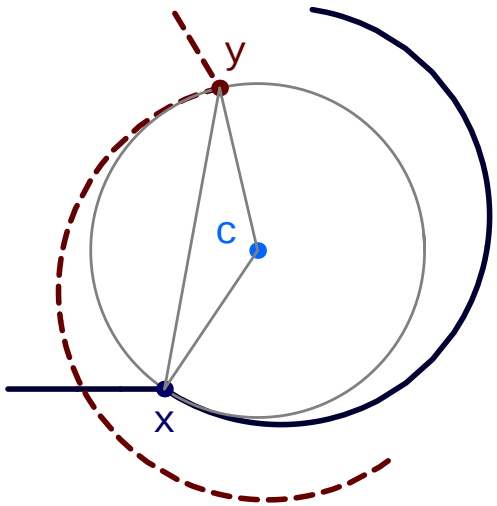


Example (d$\mathcal{L}$ formula of verification subgoal)

$$safe \wedge tangential \rightarrow [circ](safe \wedge tangential)$$

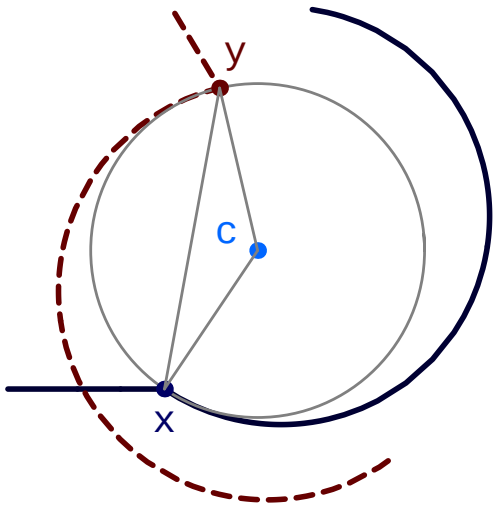
Verify Roundabout Flight with Differential Invariants

$$\overline{[x_1' = d_1, d_1' = -\omega d_2, x_2' = d_2, d_2' = \omega d_1 \ldots](x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2}$$



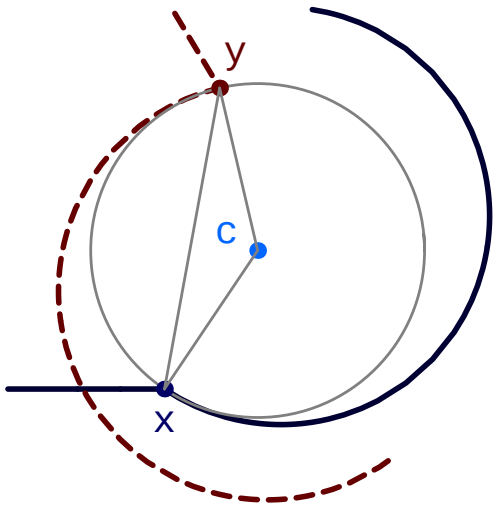
Verify Roundabout Flight with Differential Invariants

$$\frac{\frac{\partial\|x-y\|^2}{\partial x_1}x_1' + \frac{\partial\|x-y\|^2}{\partial y_1}y_1' + \frac{\partial\|x-y\|^2}{\partial x_2}x_2' + \frac{\partial\|x-y\|^2}{\partial y_2}y_2' \geq \frac{\partial p^2}{\partial x_1}x_1' \ldots}{[x_1' = d_1, d_1' = -\omega d_2, x_2' = d_2, d_2' = \omega d_1 \ldots](x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2}$$



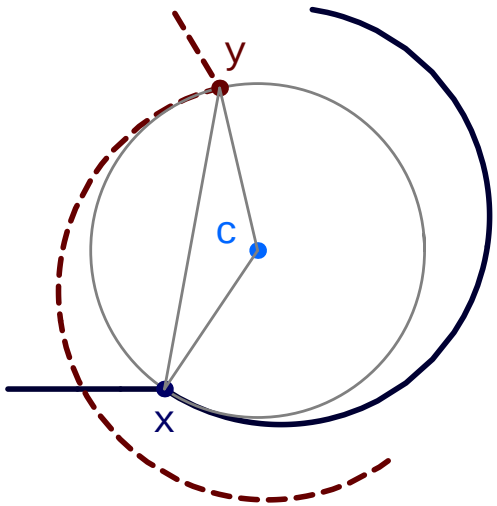
Verify Roundabout Flight with Differential Invariants

$$\frac{\frac{\partial \|x-y\|^2}{\partial x_1} x_1' + \frac{\partial \|x-y\|^2}{\partial y_1} y_1' + \frac{\partial \|x-y\|^2}{\partial x_2} x_2' + \frac{\partial \|x-y\|^2}{\partial y_2} y_2' \geq \frac{\partial p^2}{\partial x_1} x_1' \ldots}{[x_1' = d_1, d_1' = -\omega d_2, x_2' = d_2, d_2' = \omega d_1 \ldots](x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2}$$



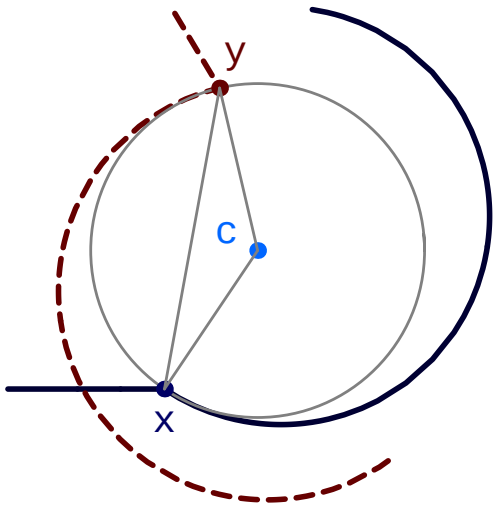
Verify Roundabout Flight with Differential Invariants

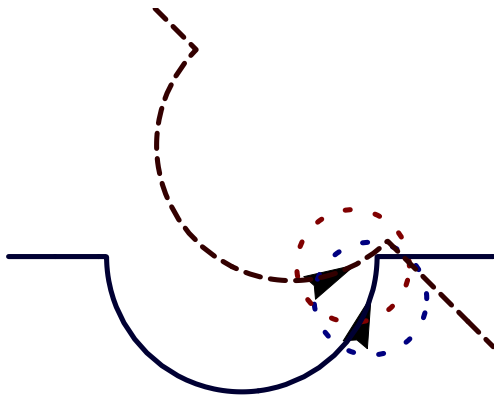
$$\frac{\frac{\partial\|x-y\|^2}{\partial x_1}d_1 + \frac{\partial\|x-y\|^2}{\partial y_1}e_1 + \frac{\partial\|x-y\|^2}{\partial x_2}d_2 + \frac{\partial\|x-y\|^2}{\partial y_2}e_2 \geq \frac{\partial p^2}{\partial x_1}d_1 \ldots}{[x_1' = d_1, d_1' = -\omega d_2, x_2' = d_2, d_2' = \omega d_1 \ldots](x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2}$$



Verify Roundabout Flight with Differential Invariants

$$\frac{2(x_1 - y_1)(d_1 - e_1) + 2(x_2 - y_2)(d_2 - e_2) \geq 0}{\frac{\partial \|x-y\|^2}{\partial x_1} d_1 + \frac{\partial \|x-y\|^2}{\partial y_1} e_1 + \frac{\partial \|x-y\|^2}{\partial x_2} d_2 + \frac{\partial \|x-y\|^2}{\partial y_2} e_2 \geq \frac{\partial p^2}{\partial x_1} d_1 \ldots}{[x_1' = d_1, d_1' = -\omega d_2, x_2' = d_2, d_2' = \omega d_1 \ldots](x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2}$$



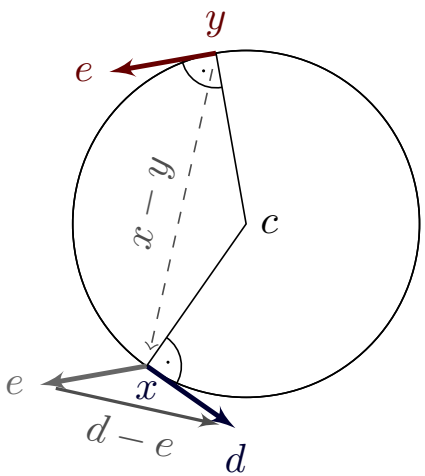


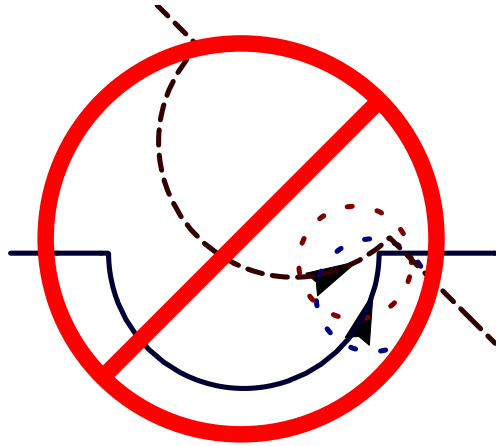
Verify Roundabout Flight with Differential Invariants

$$2(x_1 - y_1)(d_1 - e_1) + 2(x_2 - y_2)(d_2 - e_2) \geq 0$$

$$\frac{\partial \|x-y\|^2}{\partial x_1} d_1 + \frac{\partial \|x-y\|^2}{\partial y_1} e_1 + \frac{\partial \|x-y\|^2}{\partial x_2} d_2 + \frac{\partial \|x-y\|^2}{\partial y_2} e_2 \geq \frac{\partial p^2}{\partial x_1} d_1 \ldots$$

$$[x_1' = d_1, d_1' = -\omega d_2, x_2' = d_2, d_2' = \omega d_1 \ldots](x_1 - y_1)^2 + (x_2 - y_2)^2 \geq p^2$$





$$[d_1' = -\omega d_2, e_1' = -\omega e_2, x_2' = d_2, d_2' = \omega d_1 ..] d_1 - e_1 = -\omega(x_2 - y_2)$$

Verify Roundabout Flight with Differential Invariants

$$\frac{2(x_1-y_1)(d_1-e_1)+2(x_2-y_2)(d_2-e_2)\geq 0}{\frac{\partial\|x-y\|^2}{\partial x_1}d_1+\frac{\partial\|x-y\|^2}{\partial y_1}e_1+\frac{\partial\|x-y\|^2}{\partial x_2}d_2+\frac{\partial\|x-y\|^2}{\partial y_2}e_2\geq\frac{\partial p^2}{\partial x_1}d_1\ldots}{[x_1'=d_1,d_1'=-\omega d_2,x_2'=d_2,d_2'=\omega d_1\ldots](x_1-y_1)^2+(x_2-y_2)^2\geq p^2}$$

Proposition (Differential saturation)

F differential invariant of $[x'=\theta\wedge H]\mathcal{S}$, then

$$[x'=\theta\wedge H]\mathcal{S} \quad \text{iff} \quad [x'=\theta\wedge H\wedge F]\mathcal{S}$$

$$[d_1'=-\omega d_2,e_1'=-\omega e_2,x_2'=d_2,d_2'=\omega d_1..]d_1-e_1=-\omega(x_2-y_2)$$

Verify Roundabout Flight with Differential Invariants

$$\frac{2(x_1-y_1)(-\omega(x_2-y_2))+2(x_2-y_2)\omega(x_1-y_1)\geq 0}{2(x_1-y_1)(d_1-e_1)+2(x_2-y_2)(d_2-e_2)\geq 0}$$
$$\frac{\frac{\partial\|x-y\|^2}{\partial x_1}d_1+\frac{\partial\|x-y\|^2}{\partial y_1}e_1+\frac{\partial\|x-y\|^2}{\partial x_2}d_2+\frac{\partial\|x-y\|^2}{\partial y_2}e_2\geq \frac{\partial p^2}{\partial x_1}d_1\ldots}{[x_1'=d_1,d_1'=-\omega d_2,x_2'=d_2,d_2'=\omega d_1\ldots](x_1-y_1)^2+(x_2-y_2)^2\geq p^2}$$

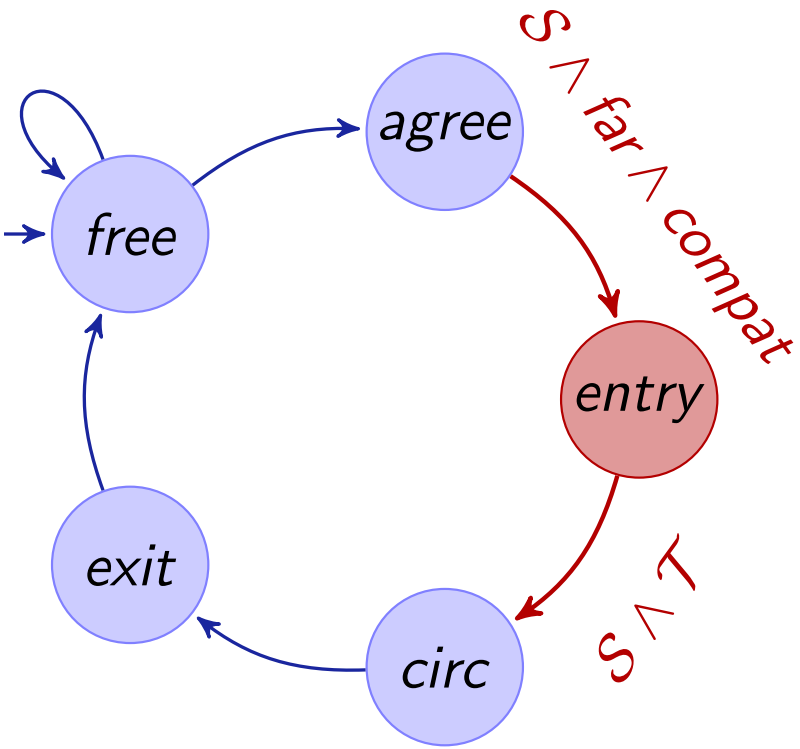
Proposition (Differential saturation)

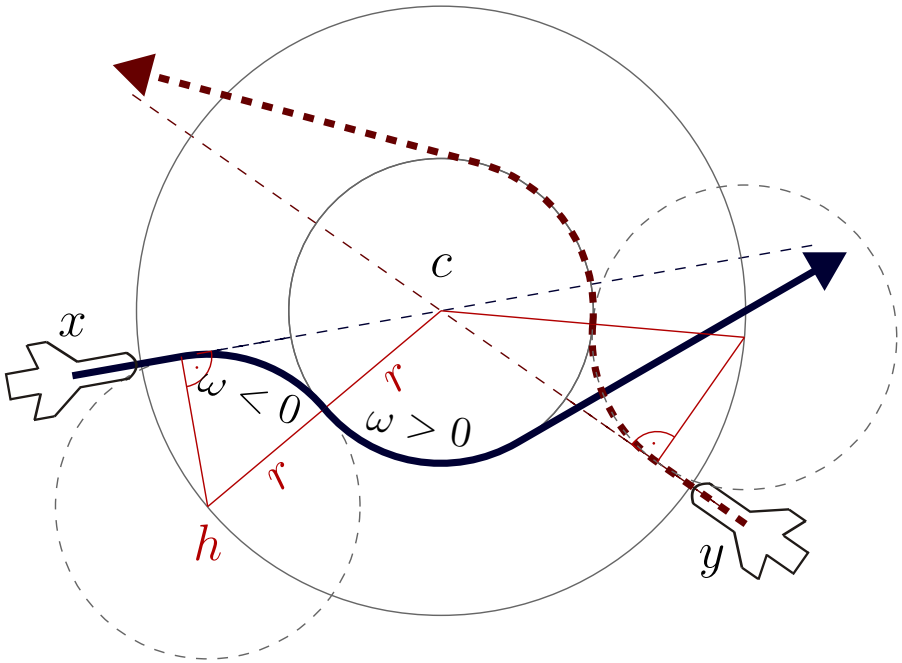
F differential invariant of $[x'=\theta\wedge H]\mathcal{S}$, then

$$[x'=\theta\wedge H]\mathcal{S} \quad \text{iff} \quad [x'=\theta\wedge H\wedge F]\mathcal{S}$$

$$[d_1'=-\omega d_2, e_1'=-\omega e_2, x_2'=d_2, d_2'=\omega d_1..]d_1-e_1=-\omega(x_2-y_2)$$

Flyable Roundabout Maneuver: Entry

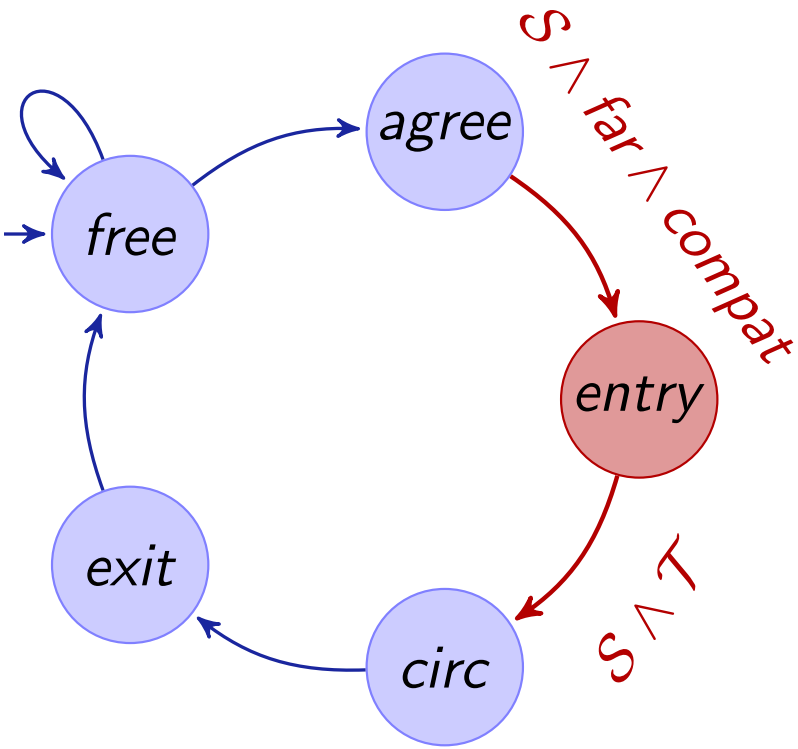


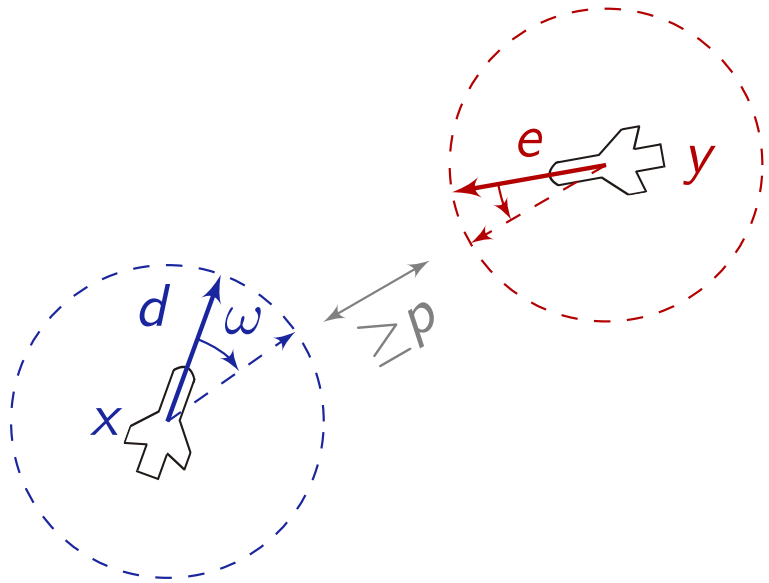


Example (d$\mathcal{L}$ formula of verification subgoal: reach tangential)

$$(r\omega)^2 = \|d\|^2 \wedge \|x-c\| = \sqrt{3}r \wedge \exists\lambda{\geq}0\,(x+\lambda d = c) \wedge$$
$$\|h-c\| = 2r \;\wedge\; d = -\omega(x-h)^{\perp}$$
$$\rightarrow\; [\mathcal{F}(-\omega) \wedge \|x-c\| \geq r]\,(\|x-c\| \leq r \rightarrow d = \omega(x-c)^{\perp})$$

Flyable Roundabout Maneuver: Entry

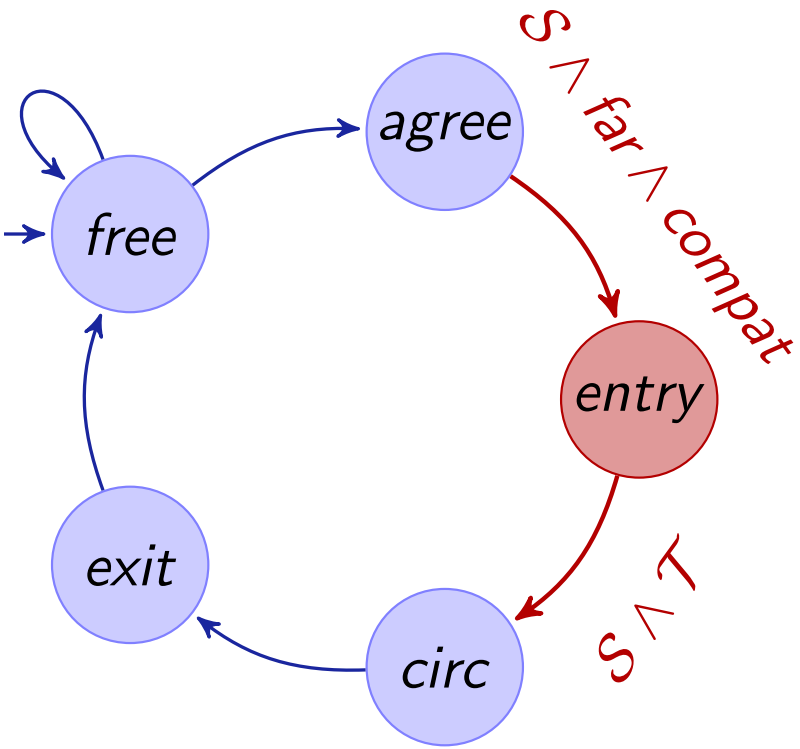


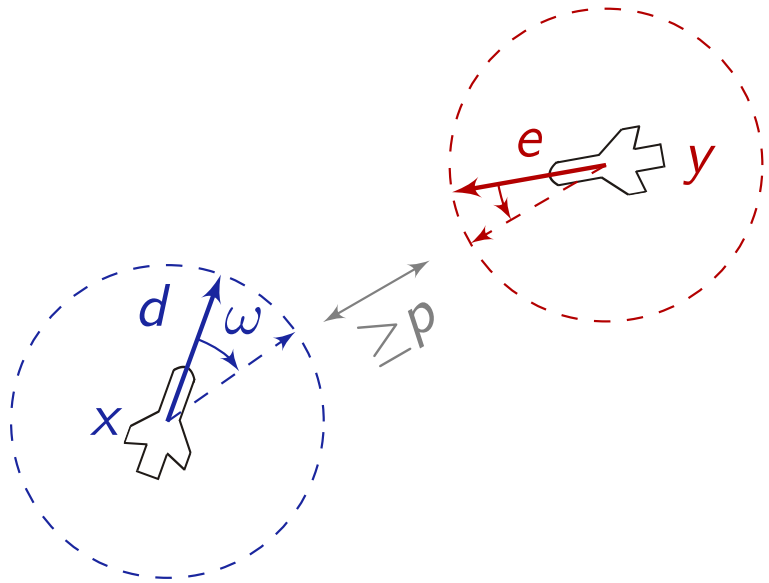


Example (d$\mathcal{L}$ formula of verification subgoal: stay separate)

$$\|x-y\| \geq \sqrt{2}(p+2bT) \wedge p \geq 0 \wedge \|d\|^2 \leq \|e\|^2 \leq b^2 \wedge b \geq 0 \wedge T \geq 0 \rightarrow [entry](\|x-y\| \geq p)$$

Flyable Roundabout Maneuver: Entry

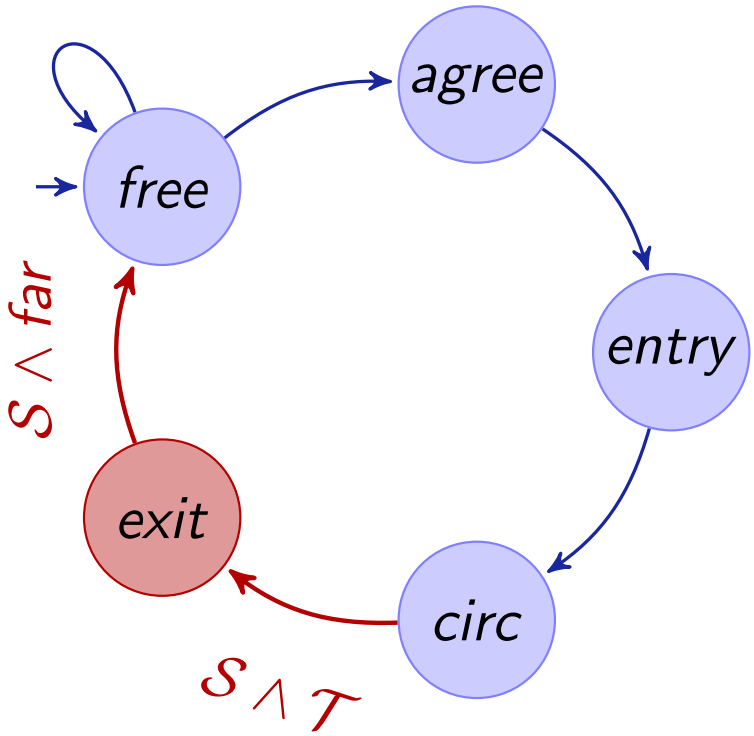


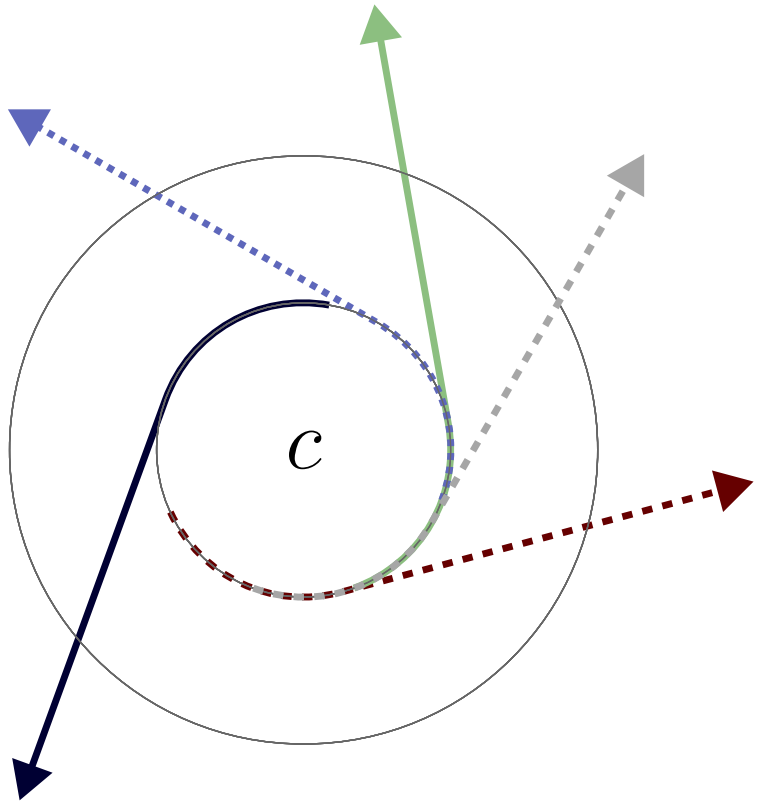


Example (d$\mathcal{L}$ formula of verification subgoal: limited progress)

$$x = z \wedge \|d\|^2 \leq b^2 \wedge b \geq 0$$
$$\rightarrow [\tau := 0;\ \exists \omega\, \mathcal{F}(\omega) \wedge \tau' = 1]\,(\|x - z\|_\infty \leq \tau b)$$

Flyable Roundabout Maneuver: Exit

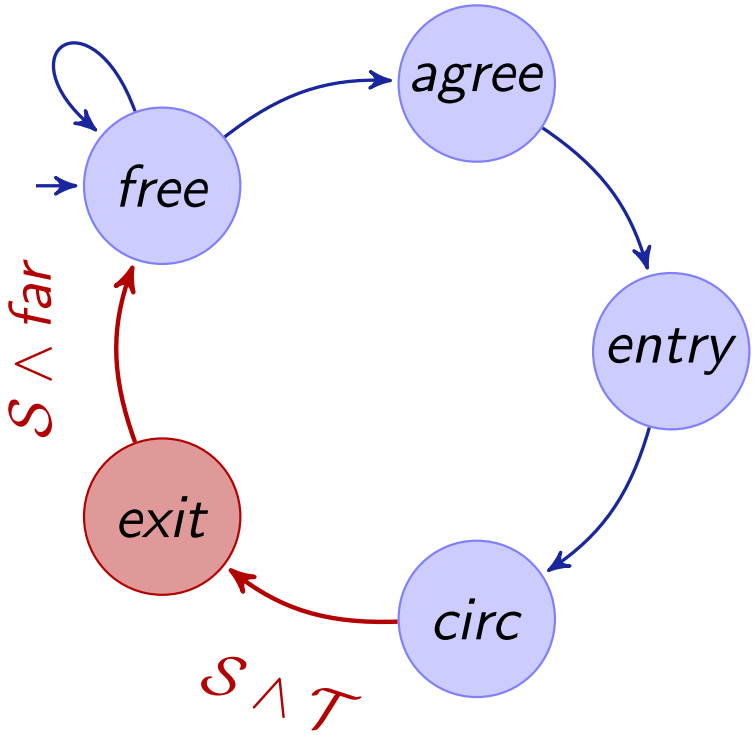


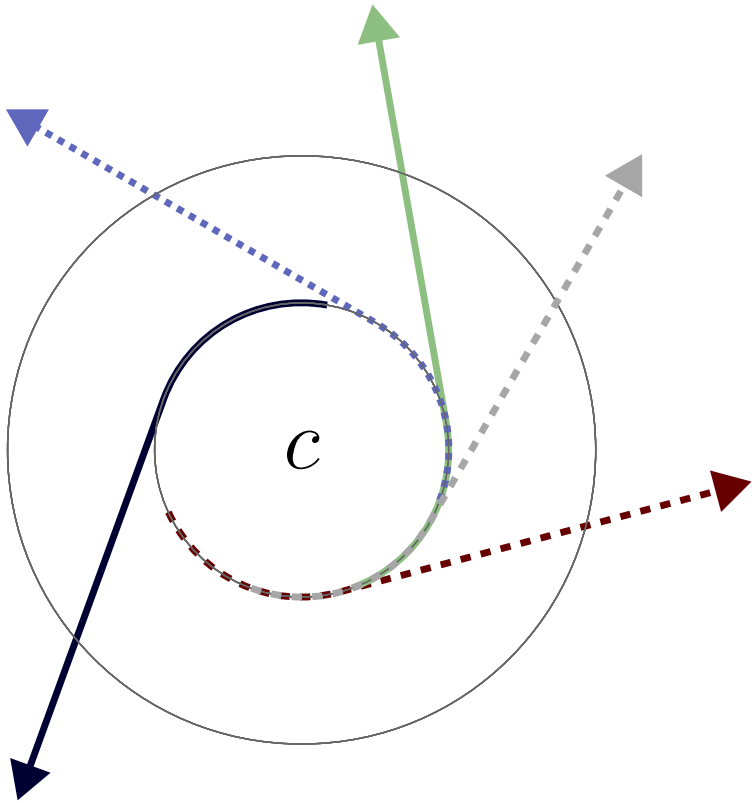


Example (d$\mathcal{L}$ formula of verification subgoal: separated exit)

$$\mathcal{T} \wedge \|x - y\|^2 \geq p^2 \rightarrow [x' = d \wedge y' = e](\|x - y\|^2 \geq p^2)$$

Flyable Roundabout Maneuver: Exit

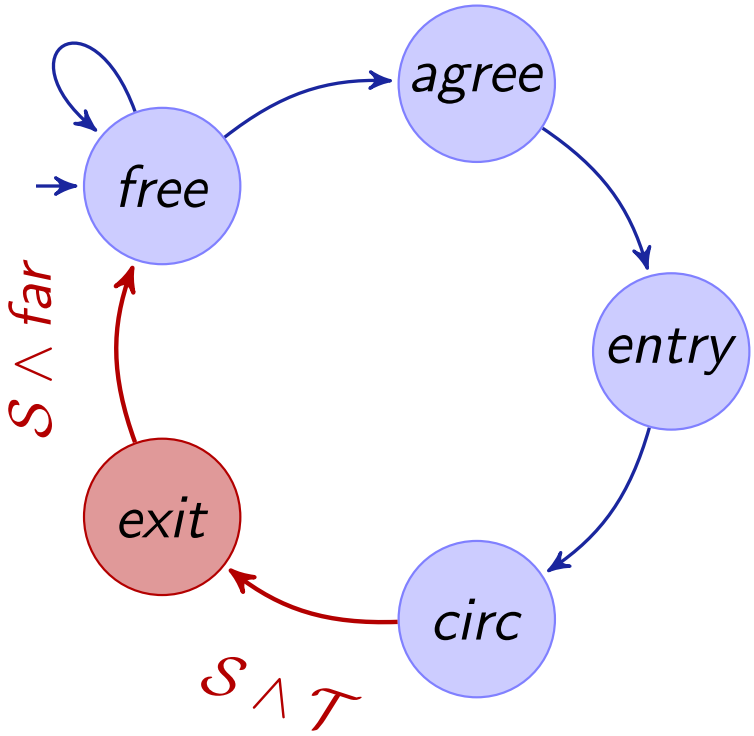


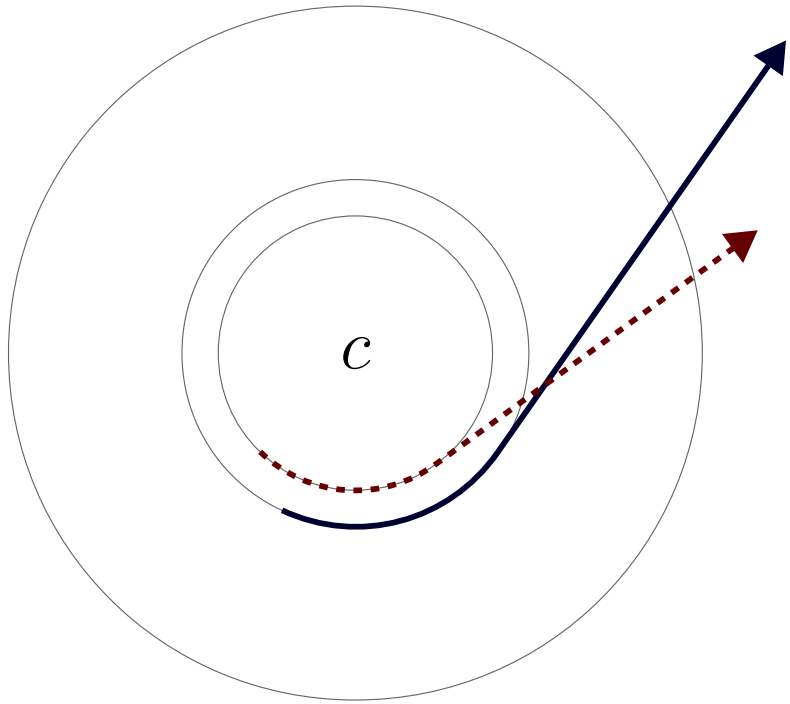


Example (d$\mathcal{L}$ formula of verification subgoal: separate directions)

$$\mathcal{T} \wedge \|x - y\|^2 \geq p^2 \rightarrow [x' = d;\ y' = e]\,(\|x - y\|^2 \geq p^2)$$

Flyable Roundabout Maneuver: Exit





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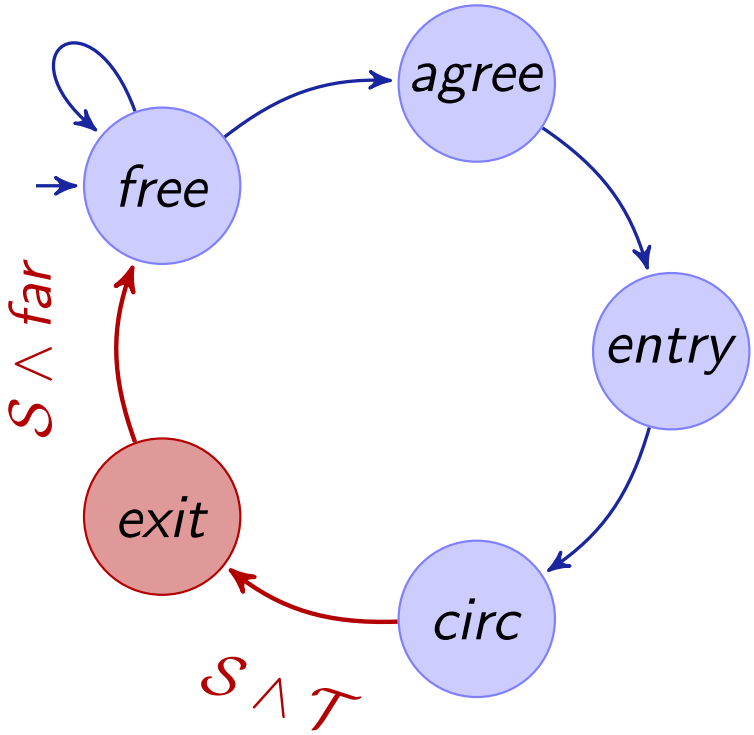
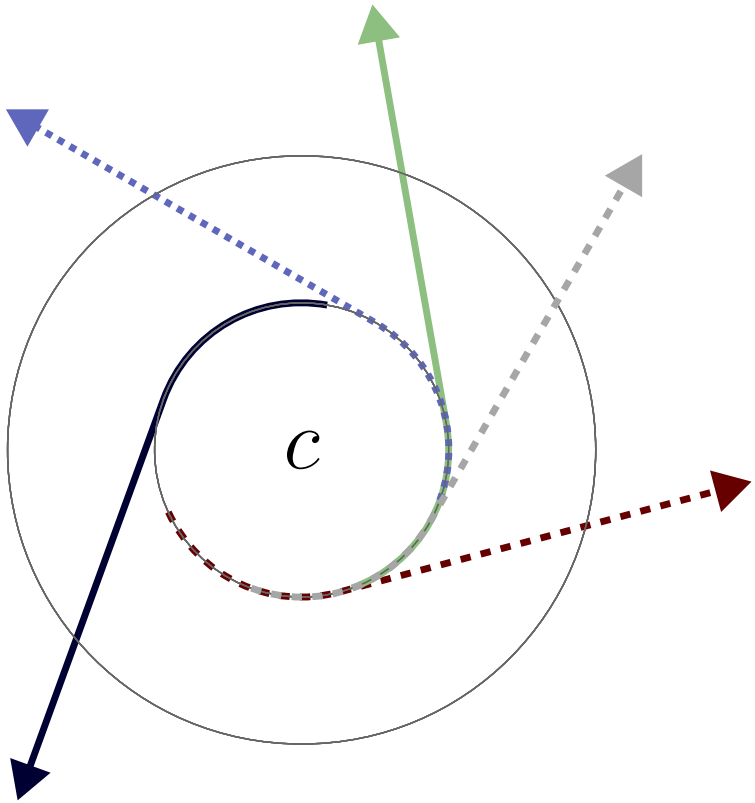
Flyable Roundabout Maneuver: Exit





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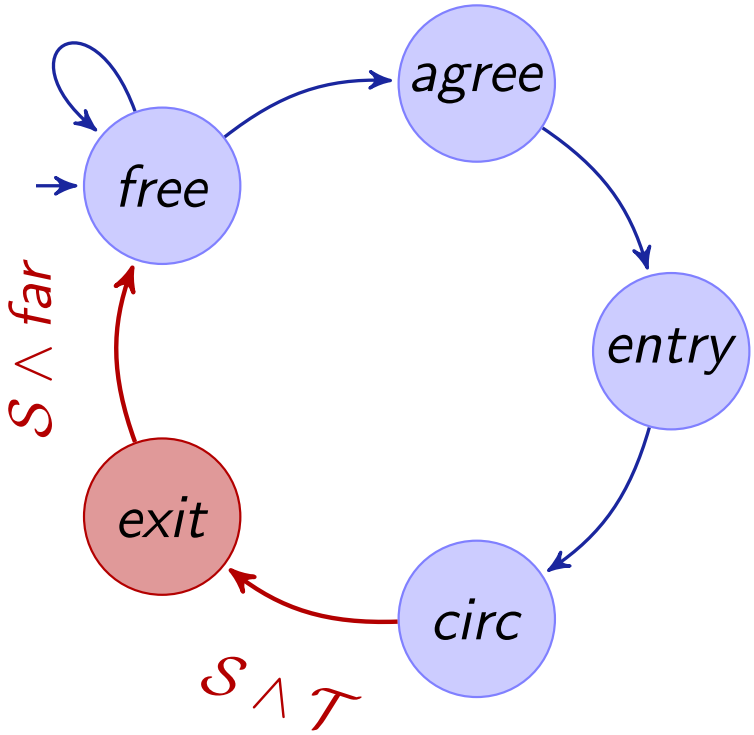
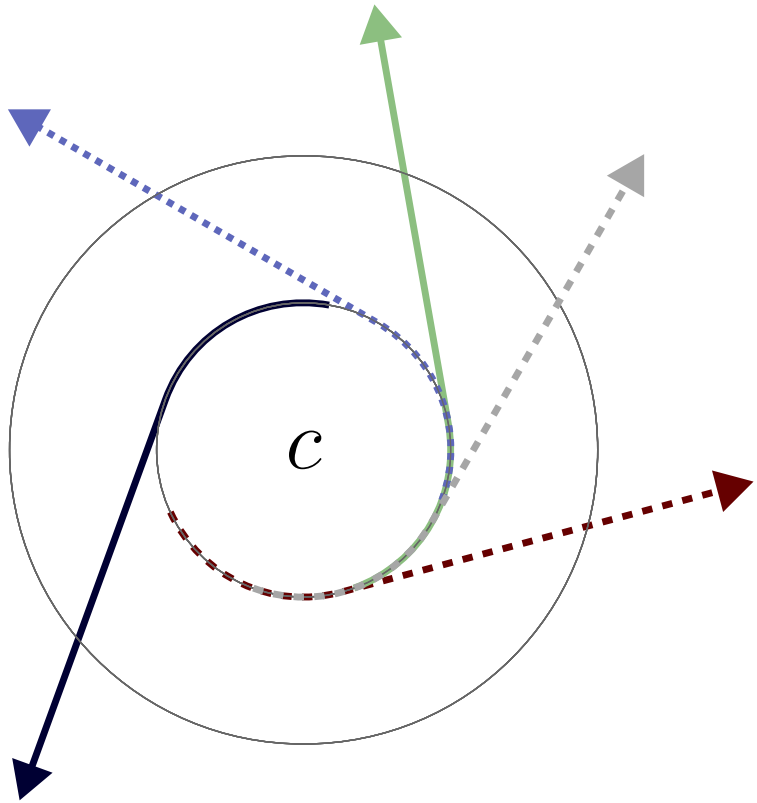


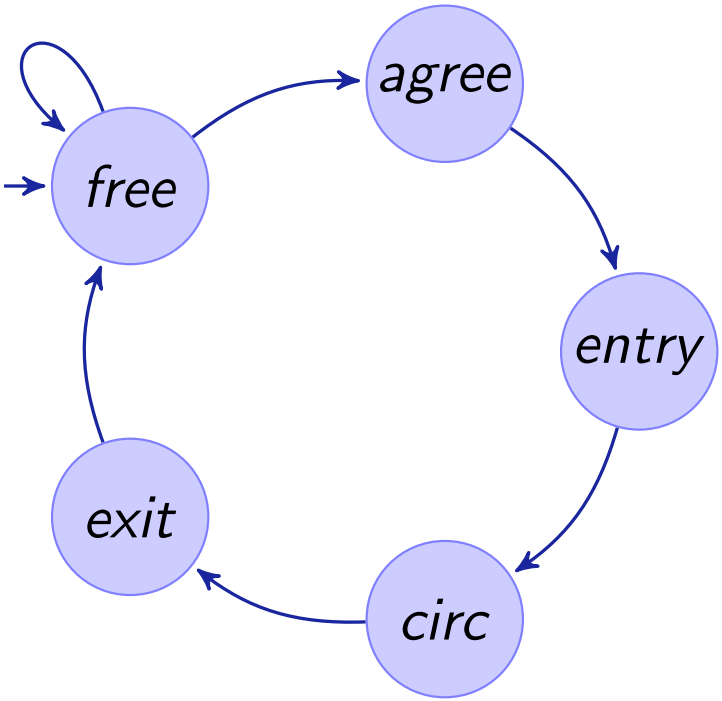


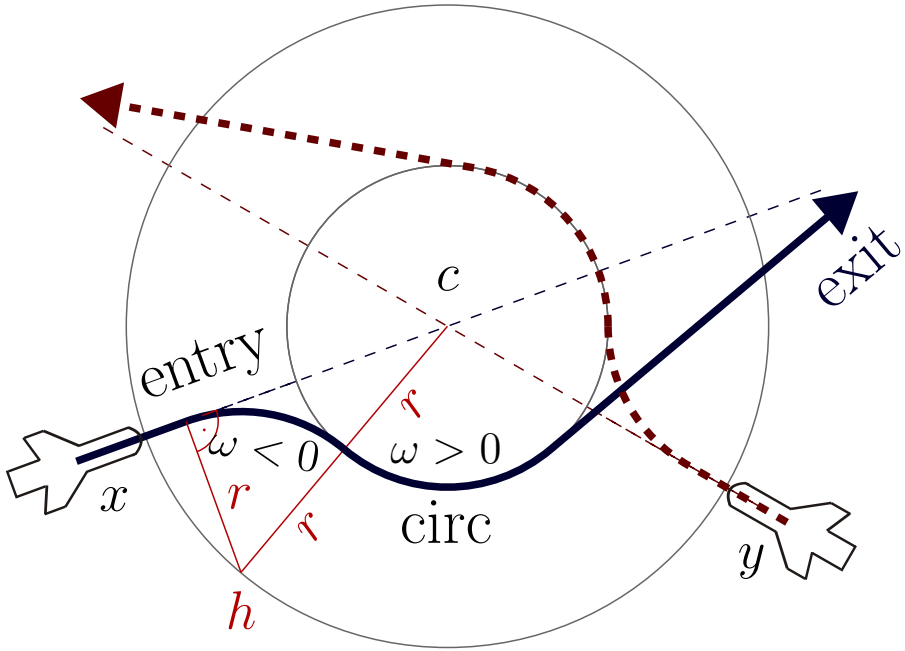
Example (d$\mathcal{L}$ formula of verification subgoal: far separability)

$$\mathcal{T} \wedge d \neq e \rightarrow \forall a \langle x' = d \wedge y' = e \rangle (\|x - y\|^2 > a^2)$$

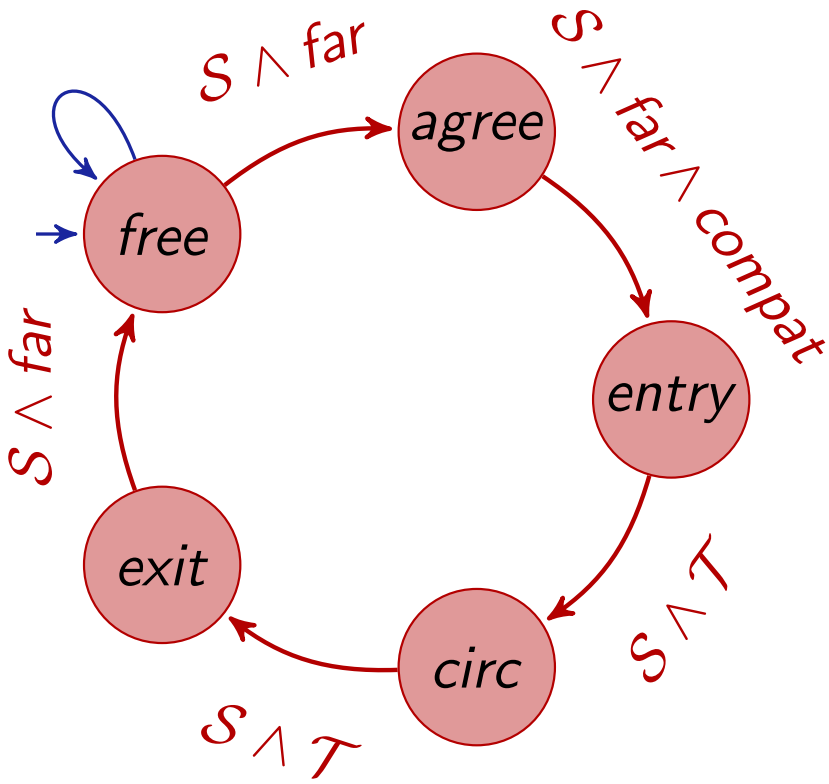
Outline

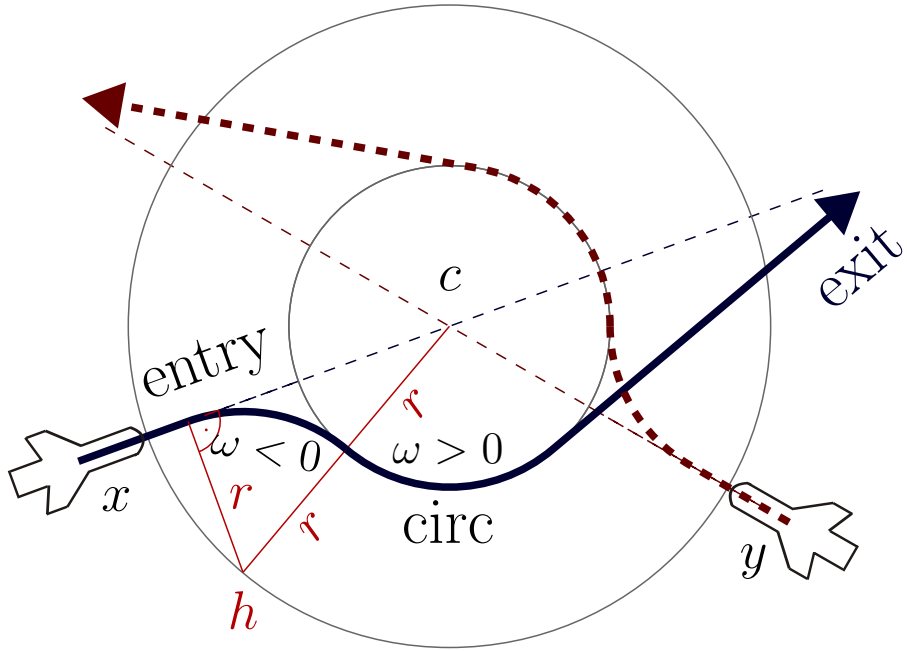
Flyable Roundabout Maneuver: Summary



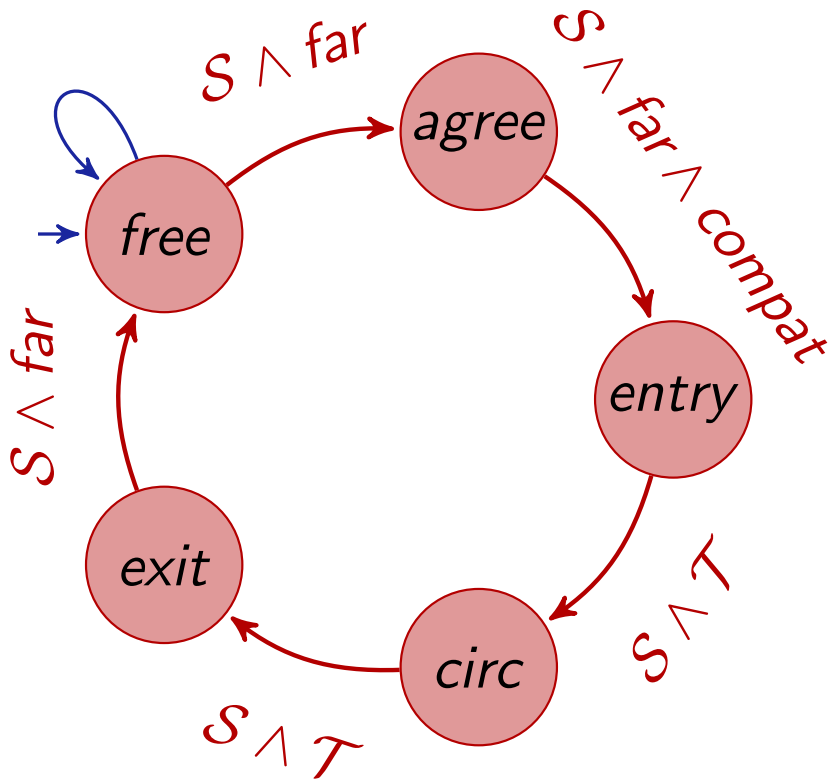


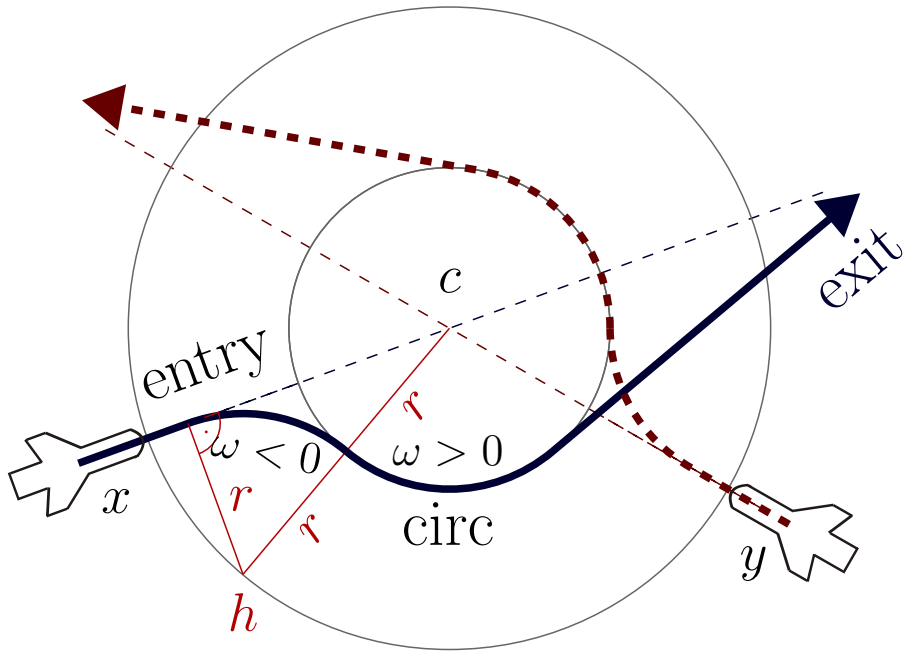
Flyable Roundabout Maneuver: Summary





Flyable Roundabout Maneuver: Summary





Theorem (Collision freedom)

FTRM is collision free:

$$\|x - y\| \geq far \wedge \ldots \rightarrow [FTRM]\|x - y\| \geq p$$

Outline

Experimental Results

Case Study	Time(s)	Mem(Mb)	Steps	Dim
tangential roundabout (2a/c)	10.4	6.8	197	13
tangential roundabout (3a/c)	253.6	7.2	342	18
tangential roundabout (4a/c)	382.9	10.2	520	23
tangential roundabout (5a/c)	1882.9	39.1	735	28
bounded maneuver speed	0.5	6.3	14	4
flyable roundabout entry*	10.1	9.6	132	8
flyable entry feasible*	104.5	87.9	16	10
flyable entry circular	3.2	7.6	81	5
limited entry progress	1.9	6.5	60	8
entry separation	140.1	20.1	512	16
mutual negotiation successful	0.8	6.4	60	12
mutual negotiation feasible*	7.5	23.8	21	11
mutual far negotiation	2.4	8.1	67	14
simultaneous exit separation*	4.3	12.9	44	9
different exit directions	3.1	11.1	42	11

Outline

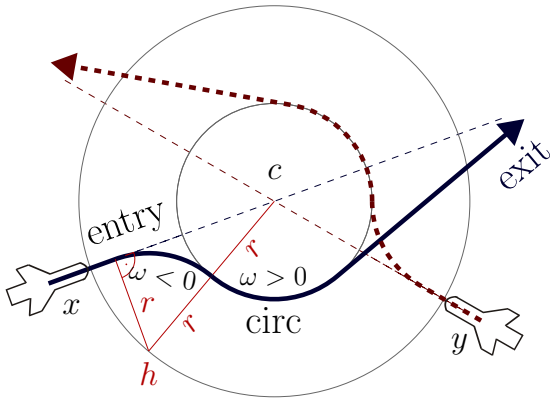
Future Work

- Scaling verification technology
- Scaling air traffic control scenarios
- Relax remaining modeling assumptions (e.g., synch)
  Proof structure is general but computational complexity challenging
- Develop and verify other entry procedure, maneuver choices, ...

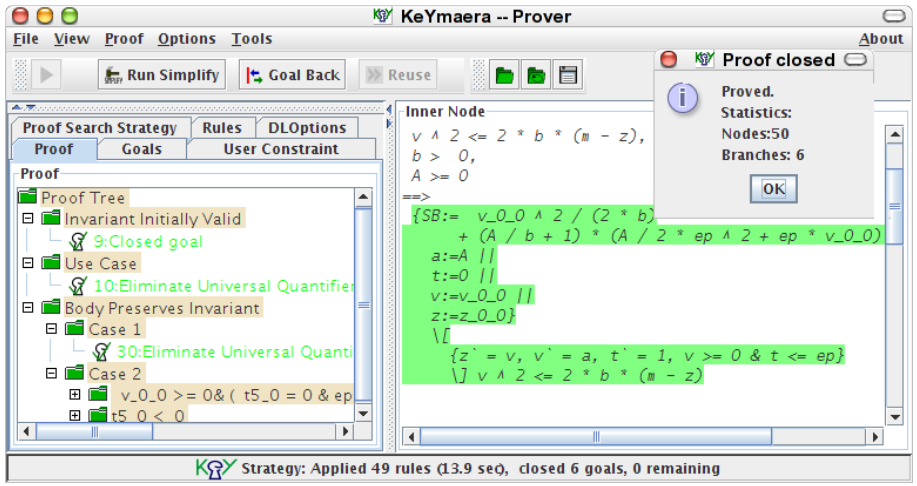
Conclusions

differential dynamic logic

$$d\mathcal{L} = \text{DL} + \text{HP}$$



- Real aircraft follow smooth curves
- Geometric intuition may mislead
- Flyable Roundabout Maneuver
- Verification in logic $d\mathcal{L}$
- Differential invariants instead of reachability along solutions
- Formal verification can scale to real aircraft maneuvers!



KeYmaera